The History of Logarithms: A glimpse of some highlights
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Martin Flashman
Department of Mathematics
Humboldt State University
flashman@humboldt.edu

Abstract

Most students learn about logarithms in intermediate algebra and elementary functions using exponential functions and the concept of an inverse function.

The early history of logarithms had some less obvious (from today's viewpoint) origins related to geometric and arithmetic rates of change and finding areas related to hyperbolas.

Outline

I will examine some of this early history of logarithms including

- Napier's 1616 original definition and tables of logarithms,
- The work of Gregoire de Saint-Vincent in 1647, and
- Newton's 1676 approach to estimating some values of natural (or hyperbolic) logarithms.

MIRIFICI

Logarithmorum
Canonis deſcriptio,

Ejuſque uſus, in utraque *Trigonometria; ut etiam in* omni Logiſtica Mathematica, *Ampliſsimi, Faciliſsimi, &* *expeditiſsimi explicatio.*

Authore ac Inventore,
IOANNE NEPERO,
Barone Merchiſtonii,
&c. Scoto.

EDINBURGI,
Ex officinâ ANDREÆ HART
Bibliopolæ, CIↃ. DC. XIV.

Part I: Napier

Napier: A Description of the Admirable Table of Logarithms

Translated from Latin to English by Edward Wright (1616).

Napier from http://www.johnnapier.com/table_of_logarithms_001.htm

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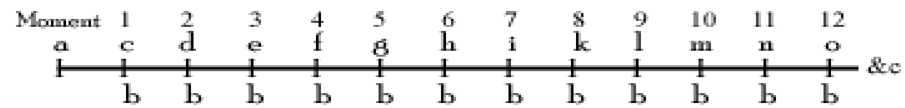
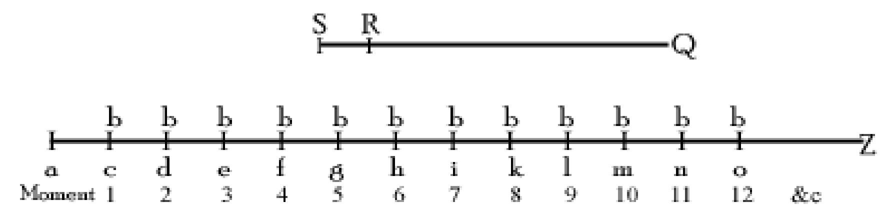
(Text in html from www.johnnapier.com)

Excerpt: Definition of logarithm

6. Definition The Logarithme therfore of any sine is a number very neerely expressing the line, which increased equally in the meane time, whiles the line of the whole sine decreased proportionally into that sine, both motions being equal-timed, and the beginning equally swift.





Napier: Proposition 1.

Proposition 1.

The Logarithmes of Proportionall numbers and quantities are equally differing.

Napier comment on details.

- An Admonition.
- Hitherto we haue shewed the making and symptomes of Logarithmes; **Now by what kinde of account or method of calculating they may be had, it should here bee shewed. But** because we do here set down the whole Tables, and all his Logarithmes with their Sines to euery minute of the quadrant: therfore passinf ouer the doctrine of making Logarithmes, til a fitter time, **we make haste to the vse of them: that the vse and profit of the thing being first conceiued,** the rest may please the more, being set forth hereafter, or else displease the lesse, being buried in silence.

•A table (based on 100) that demonstrates the idea.

A table (based on 100)

How would one use Napier's Tables:

Example: The rule of 3.

Suppose a/b = c/d.

Given any three of these values, find the fourth.

Napier's Theorem: If a/b=c/d then

NOG(a)- NOG(b) = NOG(c)-NOG(d)

Making Napier logarithm tables.

Making Napier logarithm tables. (MS Excel)

If time permits at the end:

How do Napier logarithms compare with modern logarithms?

Part II. Gregoire de St. Vincent and hyperbolic areas.

Preface:

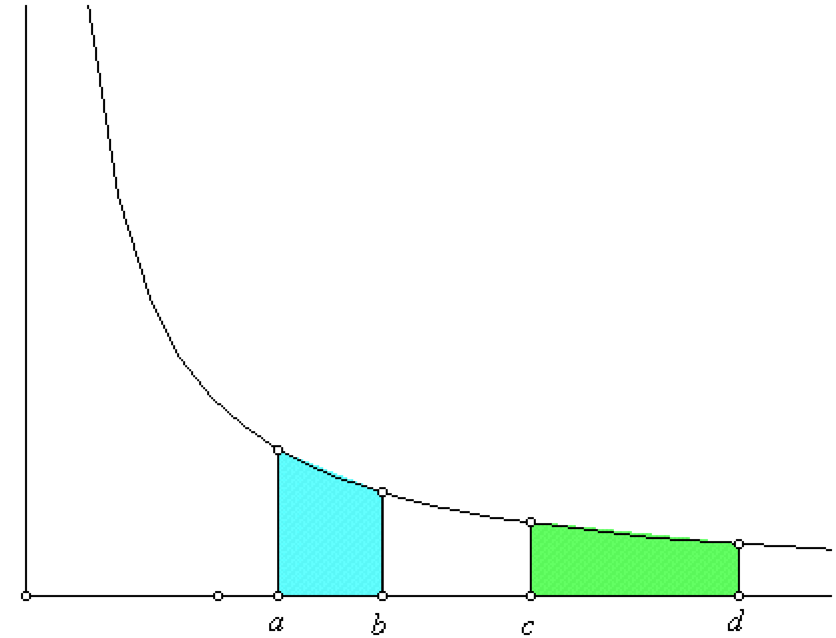
In 1637, Descartes published La Geometrie as an appendix to his Discours de la Methode.

By about 1640 the solution to the "area problem" for curves with equation $\mathbf{Y^n = aX^m}$ was known by Fermat for all integer cases except when **n = 1, m = -1**,

i.e., $\mathbf{Y = 1/X}$

Hyperbolic Areas

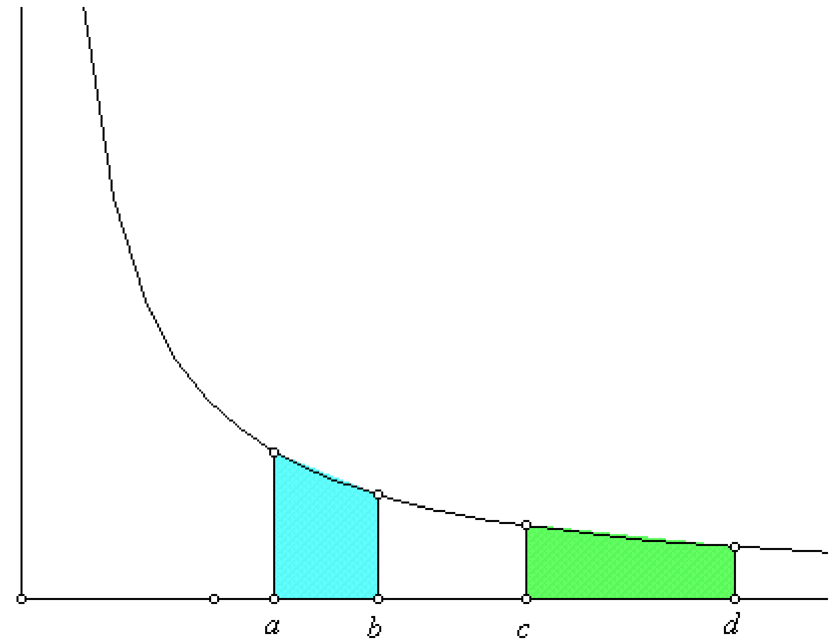
- In 1647, Gregoire de St. Vincent showed: If a/b=c/d then **the area under the hyperbola above the interval [a,b] was equal to the area under the hyperbola above the interval [c,d].**



The Hyperbolic Logarithm

In 1649 Alfonso Antonio de Sarasa recognized this feature in Gregoire's work and connected it to the properties of logarithms.

In particular he recognized the additive property of logarithms: **that if areas are all measured using a = 1, then the area determined by a product of two numbers, rs, is equal to the sum of the areas determined by r and s separately.**



Part III . Newton's computations of hyperbolic logarithms.

- In 1676 Newton wrote in a letter to Henry Oldenburg on some of his applications of series to estimating areas, in particular in estimating areas for the hyperbolic logarithm.
- This work was later clarified in Of the Method of Fluxions and Infinite Series which was published posthumously in 1737, ten years after Newton's death.

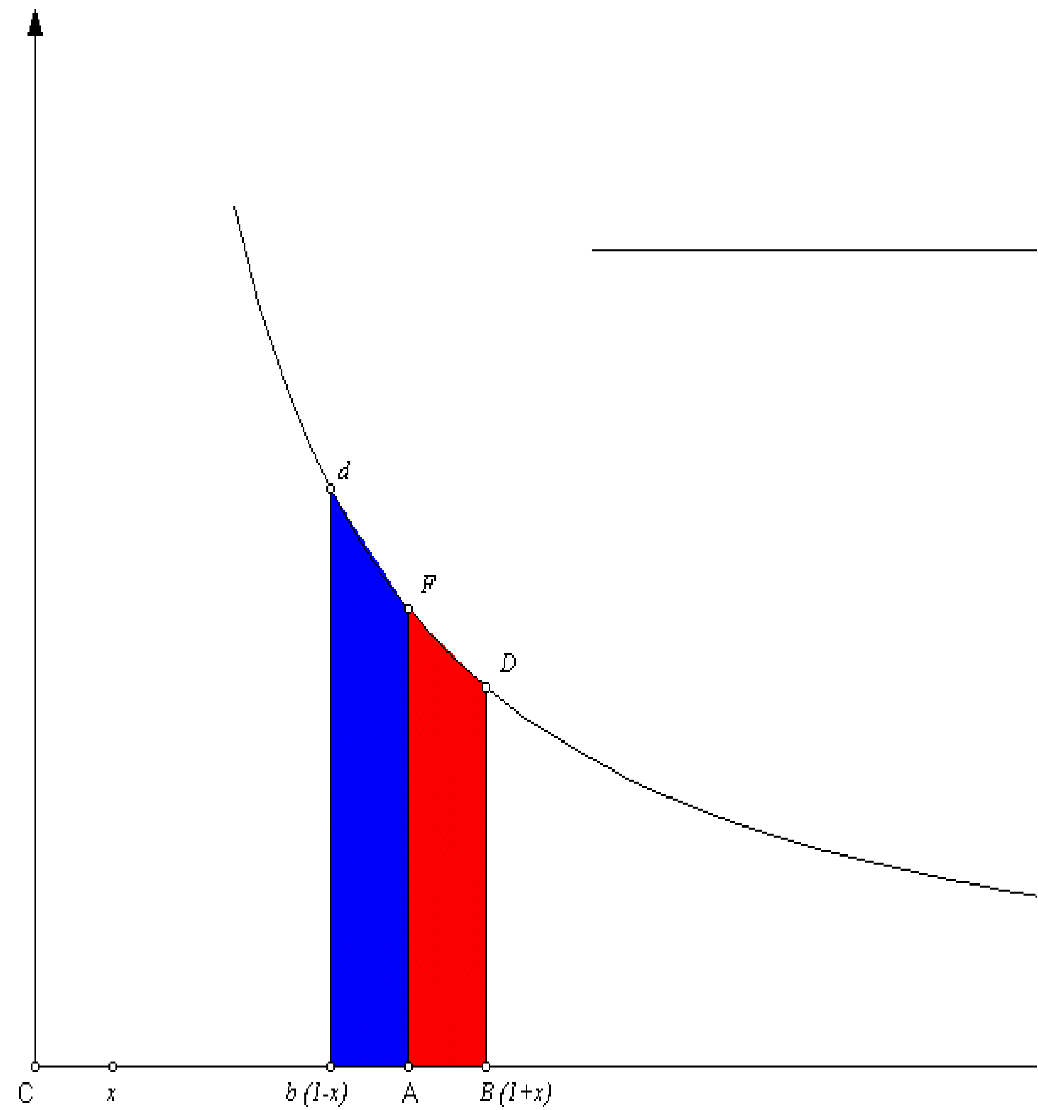
Newton estimates the Hyperbolic Log

Newton considers symmetrically located points on the main axis, 1+x and 1-x with x>0 and their related reciprocals.

He then uses two integrals related to the geometric series to determine the related areas,

(i) between the hyperbola and above the segment [1,1+x] (red) and

(ii) between the hyperbola and above the segment [1-x, 1] (blue).



$$Area\ AFDB = \int_0^h \frac{1}{1+x}\,dx = \int_0^h 1 - x + x^2 - x^3 + \ldots = h - \frac{h^2}{2} + \frac{h^3}{3} - \frac{h^4}{4} \ldots$$

$$Area\ AFdb = \int_0^h \frac{1}{1-x}\,dx = \int_0^h 1 + x + x^2 + \ldots + x^k + .. = h + \frac{h^2}{2} + \frac{h^3}{3} + \ldots \frac{h^k}{k} + \ldots$$

These allow the estimation of the sum and difference of the two areas:

$$Total\ area\ bdDB = 2h + 2\frac{h^3}{3} + 2\frac{h^5}{5} + 2\frac{h^7}{7} + \ldots$$

$$Difference\ of\ areas\ Ad - AD = h^2 + \frac{h^4}{2} + \frac{h^6}{3} + \frac{h^8}{4} + \ldots$$

Now to find the Area of the two separate regions (and related logarithms) we take 1/2 of the difference of these results and 1/2 of the sum of these results.

- Newton uses the first eight terms with

h = .1 (and .2) to estimate

the hyperbolic log of .9 and 1.1 (.8 and 1.2).

- Go to computation web page.

- A visualization of Newton's calculations using Winplot.

The End

Another reference: Logarithms: The Early History of a Familiar Function

by Kathleen M. Clark (Florida State University) and Clemency Montelle (University of Canterbury)

http://mathdl.maa.org/mathDL/46/?pa=content&sa=viewDocument&nodeId=3495&bodyId=3845