"A Sensible Approach to Calculus: Differential Equations, Estimation, and Modelling in The Fundamental Theorem."

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What's Happening Now in The First Calculus Course

- Differential and Integral Calculus with a variety of theory and applications.
- Differential Calculus: The derivative and applications- graphing, extremes, rates, Newton's method, mixing continuity and differentiability in theory, some slight mention of differential equations, THEN...
- Integral Calculus! Area, area, area, then Magic!
- The Fundamental Theorems of Calculus

What happens in "Calc II"

- Then Applications and methods of integration
- THEN…
- Sequences and Sums of Numbers: the Theory and Tests of Convergence
- Power series
- Applications: Series used for
  - Estimation of Numbers: e, pi, sqrt,…
  - Estimation of Definite Integrals.
  - Solution of Differential Equations.

What's Happening Now in The First Calculus Course

Critique

- **Little motivation for series from previous work despite**
  - **Newton's method.**
  - **Estimates of Definite Integrals.**
  - **Solutions to Differential Equations.**
- **Lengthy convergence testing delays connection with the previous work.**
- **Unclear statement of what is fundamental.**

Making Sense of The First Calculus Course

- Experience- two years of teaching 2nd semester of Calculus with students who had 1st semester with another instructor.
- Need for a better approach for all of first year.
- Focus attention on three basic themes for the entire course:
  - **Differential Equations,**
  - **Estimation, and**
  - **Mathematical Modeling**
- **Editorial *A Sensible Calculus* The UMAP Journal (1990) 11 : 93-96.**

Make Connections

- Related Rates and Implicit differentiation involve "differential equations"
- Work on graphing using the derivative involves making qualitative inferences about a function from information about its derivative.
- Applications of the Mean Value Theorem suggest uniqueness of solution to IVP.
- Review of previous work on estimates using the differential (linear estimator).

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Two Forms of the Fundamental Theorem of Calculus

Evaluation Form

If *f* is continuous and $G'(x) = f(x)$ for all x .... then

$$\int_a^b \mathbf{f(x)\,dx = G(b) - G(a).}$$

Derivative Form (Barrow's Theorem)

If *f* is continuous and $G(t) = \int_a^t f(x)\,dx$ then

G is a differentiable function and $\mathbf{G'(t) = f(t)}$.

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Fundamental Theorem of Calculus
Evaluation Form

If f is continuous and $G'(x) = f(x)$ for all x ….
then $\int_a^b f(x)\,dx = G(b) - G(a)$.

Interpretation:

$G(x)$ is a position function for a moving object which has its velocity at time x given by $f(x)$.

$\int_a^b f(x)\,dx$ represents the net change in position of the object from time a to time b.

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The Fundamental Theorem of Calculus
Derivative Form (Barrow's Theorem)

If f is continuous and $G(t) = \int_a^t f(x)\,dx$ then G is a differentiable function and $G'(t) = f(t)$.

Interpretation:

$f(x)$ is velocity of object at time x.

$G(t)$ is net change in position of object from time a to time t.

$G'(t)$ = velocity of object.

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The **FT of Calculus, DE's, and Euler's Method**

The motivation for the FT of *C* comes from estimating a solution to an Initial Value Problem, visual and numerical estimation with graphs and mapping diagrams.

Ch III.A.1. THE DIFFERENTIAL

Ch IV Differential Equations from an Elementary

V.A The Definite Integral – Connecting the definition to Euler's method and DE's.

Estimating solutions to IVP's

Initial Value Problem (IVP) :

Given $y' = f'(x) = P(x)$ and $f(a) = c$,

find exactly or estimate $f(b)$.

Connect to previous work on estimates using the differential (linear estimator).

Euler's method evolves from a progression of estimates for solving an initial value problem.

Euler's Method

- Euler's method evolves from a progression of estimates for solving an initial value problem:
- Given $y' = f'(x) = P(x,y)$ and $f(a) = c$, find or estimate $f(b)$.
  - One Step: the differential.
  - Two Equal Steps: the differential reset after first step.
  - N Equal Steps: The differential reset after each step
  - Use of spread sheets to make the estimation systematic.
- Ease of estimation of net change when $f'(x)$ depends only on x.

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The Definite Integral, DE's, and Euler's Method

The motivation for defining the definite integral comes from estimating a solution to an Initial Value Problem, visual and numerical estimation with graphs and mapping diagrams.

V.A The Definite Integral – Connecting the definition to Euler's method and DE's.

The consequences of this approach-

The FT of C makes sense.

FT of Calculus
Objective & Key Ideas

Two Objectives:

- Estimate Net Change in Distance from differential equation using Euler's method for a derivative function that depends only on x
- Measure the error in using Euler's method to estimate net change for monotonic functions.

FT of Calculus
Objective & Key Ideas

Two Key Ideas:

- When x is close to a, f(x) is approximately equal to a linear function, f(a) + f '(a)(x-a).
- As long as f is a sufficiently well behaved function there is some c between a and x where
  - f(x) = f(a) + f '(c)(x-a).

Conclusion

- With this reorganization, the treatment of the Fundamental Theorem of Calculus forms a sensible part of the first year calculus program, in a thematic approach to understanding the mathematical themes:

- **Differential Equations,**
- **Estimation, and**
- **Mathematical Modeling.**

The End.

Questions

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/senscalca_x.html

Abstract

- Introduce three themes for a sensible calculus program: **Differential Equations, Estimation, and Modelling.**
- Illustrate how these themes can illuminate the first year of calculus by considering the **Fundamental Theorem of Calculus** from a sensible view of
  - DE's and
  - estimations using Euler's method
  - interpreted in a variety of modelling contexts.