# Teaching and the Philosophy of Mathematics.

MAA Minicourse #13

January, 2008

**Part 1: Sunday, January 6, 2:15 p.m. to 4:15 p.m.**

**Part 2: Tuesday, January 8, 1:00 p.m. to 3:00 p.m.**

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# The goals of the mini-course

- **Primary:** To introduce participants to issues in the philosophy of mathematics that can be used **to illuminate classroom topics** in undergraduate courses at a variety of levels and
- **Secondary**: To provide a foundation for organizing an undergraduate **course in the philosophy of mathematics** for mathematics and philosophy students.

# The content of the mini-course

- The course will focus primarily on issues related to
  i) **the nature of the objects studied in mathematics (ontology)** and
  ii) **the knowledge of the truth of assertions about these objects (epistemology)**.
- Responses ascribed to many views **such as** Platonism, formalism, intuitionism, constructivism, logicism, structuralism, social constructivism, and empiricism will be outlined.

# Disclaimer

- This minicourse will not give a comprehensive coverage of the philosophy of mathematics.
- A selection has been made of topics that illustrate where and how the philosophy of mathematics might be useful in teaching and learning mathematics.
- There is no claim that this sample represents all of the possible approaches to the issues presented.

# What is Mathematics? B. Russell

Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.

– **Bertrand Russell**, *Mysticism and Logic (1917) ch. 4*

# Issue oriented approaches to Philosophy of Mathematics

What are **ontological issues**? ***Being***

- The nature of mathematical objects.
- The existence of mathematical objects.

What are **epistemological issues**?

- The nature of mathematical ***truth***.
- Knowledge and certainty of the status of mathematical assertions.

# What is Philosophy of Mathematics?

- **Ontology** for Mathematics: "Being"
- Ontology studies the nature of the objects of mathematics.
  "What we are talking about."
  - What is a **number**?
  - What is a **point**? **line**?
  - What is a **set**?
  - In what sense do these objects exist?

# What is Philosophy of Mathematics?

- Epistemology for Mathematics: “Knowing”
- Epistemology studies the acquisition of knowledge of the truth of a mathematical statement.
  “whether what we are saying is true.”
  - Does knowledge come from experience and evidence?
  - Does knowledge come from argument and proof?
  - Is knowledge relative or absolute?

# (Simple) View of Philosophy of Mathematics circa 1980 (Mention E. Snapper article)

- Platonism
- Formalism
- Logicism
- Intuitionism

# Platonism:

- Mathematical objects are real but abstract entities. Knowledge of these objects and truth about these objects is absolute and discovered, then justified by logical argument.
- Not verifiable directly!

# Formalism

- The objects of mathematics are the formal relationships in a formal language (of symbols or words) that are connected and known through formal definitions and arguments. (Hilbert)
- Validated by consistency. Not adequate for "all mathematics".

# Logicism

- Mathematical objects are a special kind of logical object. All mathematics can be reduced to a part of logic. (Frege, Russell)
- Reduction does not remove philosophical issues. Failure in adequacy of reduction program.

# Intuitionism

- Mathematical objects are concepts constructed  and known from a few "a priori" objects and methods that use clear and finitistic definitions and arguments. (Brouwer)
- Restriction removes many established results.

# Views of Philosophy of Mathematics More Recent

- Constructivism (derived from Intuitionism)
- Structuralism
- Fictionalism
- Naturalism-Empiricism
- Social Constructivism

# Constructivism

- Mathematical objects are constructed and statements about these objects are justified through processes that are consistent with the primitive notion of a finite process (algorithm). [Bishop]
- A realization of Intuitionist epistemology with a more open (vague?) ontology.

# Structuralism

- Mathematics is a study of Structures, through the development of theories which describe structures. "consists of places that stand in structural relations to each other. Thus, derivatively, mathematical theories describe places or positions in structures. But they do not describe objects." Systems are instances of structures (models?).
  - *Ante rem:* Structures are abstract entities (Platonic)
  - *In rebus* (Nominalist) Structures exist only through their instances in concrete physical systems.

# Naturalism-Empiricism

- Mathematics is a body of knowledge of the same type as knowledge in the physical/natural sciences. [Platonist]
- Empiricism in the sciences, appropriately understood, provides a philosophical foundation for mathematics. [Quine, Putnam]
  - Empiricism demands an ontological commitment to all and only the entities indispensable to those scientific theories that are best.
  - Our best scientific theories cannot work without mathematical entities.
  - An ontological commitment to mathematical entities is required.

# Fictionalism

- Mathematical objects are fictions- with no real existence. Mathematical statements are only true relative to fictional contexts. Consistency is a major component the reliability of statements. [A Nominalist Approach.]
  - How does one choose between fictions?
  - Why do some fictions appear more universal?

# Social Constructivism

- Science/mathematics is the "social construction of reality."
  - *Neo-Kantian social constructivism*. The adoption of a scientific paradigm *successfully* imposes a quasi-metaphysical causal structure on the phenomena scientists study.
  - *Science-as-social-process social constructivism*. The production of scientific findings is a social process subject to the same sorts of influences -- cultural, economic, political, sociological, etc. -- which affect any other social process.
  - *Debunking social constructivism*. A skeptical position. The findings of work in the sciences are determined exclusively, or in large measure, not by the "facts," but instead by relations of social power within the scientific community and the broader community within which research is conducted.

# Roles for Philosophy in Teaching and Learning

- For the Teacher/Mentor (T/M)
  - Awareness of issues can alert the T/M to excessively authoritarian approaches.
  - Alternative philosophical views can allow the T/M to use and/or develop alternatives to traditional approaches.
  - Philosophical issues can illuminate the value of and need for developing a variety of mathematical tools for "solving problems".

# Roles for Philosophy in Teaching and Learning

- For the Student/Learner (S/L)
  - Helps the S/L understand the context, goals, and objectives of the mathematics being studied. I.e., it helps answer such questions as:
    - Where does this fit?
    - Where is this going?
  - Why do  we study this?
  - Opens the S/L to considerations of
    - the human values and
    - assumptions made in developing and using mathematics.
  - Alerts the S/L to
    - the use of authority and
    - the value of different approaches to mathematics.

# Exploring Initial Questions

- Following are two questions presented in the preliminary assignment. They can be used to introduce and explore some philosophical issues in courses at a variety of levels.
- Consider how the questions and the related examples can be expanded or transformed to consider many aspects of the philosophy of mathematics.
- Consider how the questions and the related examples can be expanded or transformed to other mathematics topics and/or courses.

# The point of the discussion

- The philosophical issues related to the nature of the square root of two, and other numbers
  - do not have simple or easy answers
  - can shed light on how numbers are used and understood in mathematics

# End of Session I

Questions?

Comments?

Discussion?

Next Session: Epistemology

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# Notes and Materials

- I will send by e-mail links to these notes and suggestions for further reading and on-line links in the next two weeks.
- Be sure e-mail addresses and other information are correct on MAA list.

# Second Session Topics

Review of the distinctions and overlap between ontological and epistemological issues.

- Existence and uniqueness.
  - How do we justify saying we know something exists?
  - What do existence and uniqueness mean for ontology?
  - What do they mean for epistemology!
- What does truth mean?
- How do we know the truth of assertions?

# Some of the "isms" from Session I

- Platonism
- Formalism
- Logicism
- Intuitionism / Constructivism
- Structuralism
- Fictionalism
- Naturalism-Empiricism
- Social Constructivism

# Preliminary Problem Assignment

- **Consider the following questions related to issues in the philosophy of mathematics:**
- **I. What is a number?**
  In particular, what about the nature of a number allows the following examples encountered in school mathematics to qualify as being numbers?
  2, 1, 0, 3/7 , -3, sqrt(2) , sqrt(-1) , pi, e^2, e^pi, ln(2)
- **II. How does one determine the truth or falsity of a mathematical statement?**
  In particular, how does one determine the truth of the following statements encountered in school mathematics?
  The square root of 4 is a rational number.
  There is a number which when squared yields 2.
  The square root of 2 is not a rational number.
  The square root of 2 is between 1 and 2.

# Looking at two specific examples.

- Before considering the broad range of possible answers to these questions, we'll focus on two specific examples:

  - **The square root of 2.**
  - **The square root of -1.**

# The Square Root of Two

- Courses
  - Pre-calculus
  - Calculus
  - Transitional Proof Course
  - Number Theory
  - Algebra
  - Real Analysis
  - Numerical Analysis

- Questions for Open Discussion
- Ontological:
  - Definition?
  - Does it exist?
  - What is the nature of this object?
- Epistemological
  - How do we know it exists?
  - How do we know it is "between 1 and 2"
  - How do we know it is not a rational number?

# The Square Root of -1: “$i$”

- Courses
  - Pre-calculus
  - Calculus
  - Transitional Proof Course
  - Number Theory
  - Linear Algebra
  - Algebra
  - Real Analysis
  - Complex Analysis
  - Technology/CAS

- Questions for Open Discussion
- Ontological:
  - Definition?
  - Does it exist?
  - What is the nature of this object?
- Epistemological
  - How do we know $i$ exists?
  - How do we know $i$ is not a real number?
  - How do we know that the complex numbers are algebraically closed?

# Ontology of square root of 2: the usual classroom focus

- Why this number is not rational;
- How this number exists as
  - an infinite decimal,
  - a Cauchy sequence ,
  - a Dedekind cut, or
  - an element of an algebraic extension of the rational numbers.

# Philosophical issues of the nature of the square root of 2 that are usually ignored.

- How do you define the square root of two? What is the nature of this number? Alternative philosophical views:
  - Is it an abstract entity- a real object in a platonic reality?
  - Is it a measurement of a physical object?
  - Is it anything that satisfies the formalities that characterize it in a formal system for real numbers?
  - Is it an equivalence class in a set theoretic context?
  - Is it the limit of a sequence?

# How much structure is required to define or characterize the square roof of 2 as a mathematical object?

- Sets?
- Operations?
- Geometry?
- Real Number Axioms?
- Field Axioms?

# Focus on Ontology: Numbers

- Key examples for discussion: [How are these defined?]
  - What is 2?
  - What is the square root of 2?
  - What is 0?
  - What is -1?
  - What is the square root of -1?
  - What is π?

# Focus on Ontology - Sets

- Key examples for discussion:
  - Finite sets
  - The empty set
  - Infinite sets

# Focus on Ontology: Geometry (Not discussed – no time)

- Key examples for discussion:
  - Point
  - Line
  - Plane
  - Space

# Epistemology for numerical assertions

- Existence.
- Uniqueness.
- Comparison.
- Complex numerical predicates .
- Negation.
- The role of axioms and structures.

# Epistemology for set/function assertions

- Existence.
- Uniqueness.
- Comparison.
- Complex numerical predicates .
- Negation.
- The role of axioms and structures.

# Focus on Epistemology

Example: The square root of 2 is between 1 and 2.

- What this assertion mean?
- How can one know the truth of the assertion?
  - How does authority and social acceptance influence this?
  - Is this a matter of psychology and not philosophy?
  - Should empirical evidence be persuasive?
  - Is this a fact that can be ascertained without reference to the meaning and nature of the number?
  - Is this an assertion that can be proven from other assertions that are fundamental to the nature of numbers?

# Discussion

- How might you incorporate some philosophical issues in your teaching?
- What would you want to achieve by raising these issues with students at a variety of levels of mathematical sophistication?
- What further study would you want to pursue to learn more about the philosophy of mathematics?

# What to do in the future?

- Individual:
  - Read more.
  - Join POMSIGMAA?
- With others:
  - Discussion List(s)
  - Follow up discussions from this mini-course.
- Organize:
  Encourage more discussion of these issues with colleagues at your institution –seminar/speakers
  - Math Educators - Educators
  - Philosophers - Cognitive Psychologist

# References


- Robert J. Baum, Editor, Philosophy and mathematics, from Plato to the present. (San Francisco : Freeman, Cooper, 1973)
  - These readings end with Frege while including three contemporary views to give some more modern treatments.
- Benacerraf, P. & Putnam, H. (eds.),. *Philosophy of Mathematics: Selected Readings*, (Cambridge: Cambridge University Press, 2nd edition, 1983.)
  - The “bible” for much of twentieth century philosophy of Math till about 1980. Reading this will give a firm grounding on most issues of that period. Contains an extensive bibliography.
- Philip J. Davis, Reuben Hersh, The Mathematical Experience, (Boston, Birkhauser, 1981)
  - A popular book mixing history, philosophy and exposition of mathematics. Winner of the American Book Award.
- I. Grattan-Guinness, Editor, Encyclopedia of the history and philosophy of the mathematical sciences. (London ; New York : Routledge, 1994.)
  - Though primarily a historical document – this contains some good overview articles on the philosphy of mathematics.
- Dale Jacquette, editor, Philosophy of Mathemaitcs: An Anthology, (Malden, Mass. : Blackwell, 2002.)
  - An excellent sequel to B &P with many papers and articles from after 1980.
- Hugh Lehman, Introduction to the philosophy of mathematics. (Totowa, N.J. : Rowman and Littlefield, 1979.)
  - My own favorite for a text like treatment surveying most approaches to philosophy of mathematics with a reasonable presentation of the philosphical arguments.
- Michael D. Resnik, Frege and the philosophy of mathematics (Ithaca, N.Y. : Cornell University Press, 1980.)
  - An historical review of the POM at Frege’s time with a good philosophical treatment of many subsequent responses.
- Ernst Snapper, The Three Crises in Mathematics: Logicism, Intuitionism and Formalism, in *Mathematics Magazine,* Vol 52, No. 4 (Sept., 1979), pp 207-216.
  - An article that discusses the common views of many mathematicians at that time. Rather limited in seeing more than the simplest appraoches.
- Thomas Tymoczko, editor, New directions in the philosophy of mathematics : an anthology (Boston: Birkhäuser, c1986.)
  - A collection that presents some new issues in POM that result from computer proofs and other developments.

A lengthier bibliography follows based on the web references at the Stanford Encyclopedia of Philosophy.

This is not intended as a comprehensive list.

Horsten, Leon, "Philosophy of Mathematics", *The Stanford Encyclopedia of Philosophy (Winter 2007 Edition)*, Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/win2007/entries/philosophy-mathematics/>.

- Balaguer, M. 1998. *Platonism and Anti-Platonism in Mathematics*, Oxford: Oxford University Press.
- Benacerraf, P., 1965. ‘What Numbers Could Not Be’, in Benacerraf & Putnam 1983, 272-294.
- Benacerraf, P., 1973. ‘Mathematical Truth’, in Benacerraf & Putnam 1983, 403-420.
- Benacerraf, P. & Putnam, H. (eds.), 1983. *Philosophy of Mathematics: Selected Readings*, Cambridge: Cambridge University Press, 2nd edition.
- Bernays, P., 1935. ‘On Platonism in Mathematics’, in Benacerraf & Putnam 1983, 258-271.
- Boolos, G., 1971. ‘The Iterative Conception of Set’, in Boolos 1998, 13-29.
- Boolos, G., 1975. ‘On Second-Order Logic’, in Boolos 1998, 37-53.
- Boolos, G., 1985. ‘Nominalist Platonism’, in Boolos 1998, 73-87.
- Boolos, G., 1987. ‘The Consistency of Frege's Foundations of Arithmetic’ in Boolos 1998, 183-201.
- Boolos, G., 1998. *Logic, Logic and Logic*, Cambridge: Harvard University Press.
- Burge, T., 1998. ‘Computer Proofs, A Priori Knowledge, and Other Minds’, *Noûs*, 32: 1-37.
- Burgess, J. & Rosen, G., 1997. *A Subject with No Object: Strategies for Nominalistic Interpretation of Mathematics*, Oxford: Clarendon Press.
- Burgess, J., 2004. ‘Mathematics and Bleak House’, *Philosophia Mathematica*, 12: 37-53.

- Cantor, G., 1932. *Abhandlungen mathematischen und philosophischen Inhalts*, E. Zermelo (ed.), Berlin: Julius Springer.
- Carnap, R., 1950. 'Empiricism, Semantics and Ontology', in Benacerraf & Putnam 1983, 241-257.
- Chihara, C., 1973. *Ontology and the Vicious Circle Principle*, Ithaca: Cornell University Press.
- Cohen, P., 1971. 'Comments on the Foundations of Set Theory', in D. Scott (ed.) *Axiomatic Set Theory* (Proceedings of Symposia in Pure Mathematics, Volume XIII, Part 1), American Mathematical Society, 9-15.
- Colyvan, M., 2001. *The Indispensability of Mathematics*, Oxford: Oxford University Press.
- Curry, H., 1958. *Outlines of a Formalist Philosophy of Mathematics*, Amsterdam: North-Holland.
- Detlefsen, M., 1986. *Hilbert's Program*, Dordrecht: Reidel.
- Deutsch, D., Ekert, A. & Luppacchini, R., 2000. 'Machines, Logic and Quantum Physics', *Bulletin of Symbolic Logic*, 6: 265-283.
- Feferman, S., 1988. 'Weyl Vindicated: Das Kontinuum seventy years later', reprinted in S. Feferman, *In the Light of Logic*, New York: Oxford University Press, 1998, 249-283.
- Feferman, S., 2005. 'Predicativity', in S. Shapiro (ed.), *The Oxford Handbook of Philosophy of Mathematics and Logic*, Oxford: Oxford University Press, pp. 590-624.
- Field, H., 1980. *Science without Numbers: a defense of nominalism*, Oxford: Blackwell.
- Field, H., 1989. *Realism, Mathematics & Modality*, Oxford: Blackwell.
- Frege, G., 1884. *The Foundations of Arithmetic. A Logico-mathematical Enquiry into the Concept of Number*, J.L. Austin (trans), Evanston: Northwestern University Press, 1980.

- Gödel, K., 1931. ‘On Formally Undecidable Propositions in Principia Mathematica and Related Systems I’, in van Heijenoort 1967, 596-616.
- Gödel, K., 1944. ‘Russell's Mathematical Logic’, in Benacerraf & Putnam 1983, 447-469.
- Gödel, K., 1947. ‘What is Cantor's Continuum Problem?’, in Benacerraf & Putnam 1983, 470-485.
- Goodman, N. & Quine, W., 1947. ‘Steps Towards a Constructive Nominalism’, *Journal of Symbolic Logic*, 12: 97-122.
- Halbach, V. & Horsten, L., 2005. ‘Computational Structuralism’ *Philosophia Mathematica*, 13: 174-186.
- Hale, B. & Wright, C., 2001. *The Reason's Proper Study: Essays Towards a Neo-Fregean Philosophy of Mathematics*, Oxford: Oxford University Press.
- Hallett, M., 1984. *Cantorian Set Theory and Limitation of Size*, Oxford: Clarendon Press.
- Hellman, G., 1989. *Mathematics without Numbers*, Oxford: Clarendon Press.
- Hilbert, D., 1925. ‘On the Infinite’, in Benacerraf & Putnam 1983, 183-201.
- Hodes, H., 1984. ‘Logicism and the Ontological Commitments of Arithmetic’, *Journal of Philosophy*, 3: 123-149.
- Isaacson, D., 1987. ‘Arithmetical Truth and Hidden Higher-Order Concepts’, in The Paris Logic Group (eds.), *Logic Colloquium '85*, Amsterdam: North-Holland, 147-169.
- Kreisel, G., 1967. ‘Informal Rigour and Completeness Proofs’, in I. Lakatos (ed.), *Problems in the Philosophy of Mathematics*, Amsterdam: North-Holland.
- Lavine, S., 1994. *Understanding the Infinite*, Cambridge, MA: Harvard University Press.
- Linsky, B. & Zalta, E., 1995. ‘Naturalized Platonism vs. Platonized Naturalism’, *Journal of Philosophy*, 92: 525-555.
- Maddy, P., 1988. ‘Believing the Axioms I, II’, *Journal of Symbolic Logic*, 53: 481-511, 736-764.
- Maddy, P., 1990. *Realism in Mathematics*, Oxford: Clarendon Press.
- Maddy, P., 1997. *Naturalism in Mathematics*, Oxford: Clarendon Press.
- Manders, K., 1989. ‘Domain Extensions and the Philosophy of Mathematics’, *Journal of Philosophy*, 86: 553-562.
- Martin, D.A., 1998. ‘Mathematical Evidence’, in H. Dales & G. Oliveri (eds.), *Truth in Mathematics*, Oxford: Clarendon Press, pp. 215-231.
- Martin, D.A., 2001. ‘Multiple Universes of Sets and Indeterminate Truth Values’ *Topoi*, 20: 5-16.
- McGee, V., 1997. ‘How we Learn Mathematical Language’, *Philosophical Review*, 106: 35-68.

- Moore, G., 1982. *Zermelo's Axiom of Choice: Its Origins, Development, and Influence*, New York: Springer Verlag.
- Parsons, C., 1980. ‘Mathematical Intuition’, *Proceedings of the Aristotelian Society*, 80: 145-168.
- Parsons, C., 1983. *Mathematics in Philosophy: Selected Essays*, Ithaca: Cornell University Press.
- Parsons, Ch. 1990: The Structuralist View of Mathematical Objects. Synthese 84(1990), p. 303-346.
- Pour-El, M., 1999. ‘The Structure of Computability in Analysis and Physical Theory’, in E. Griffor (ed.), *Handbook of Computability Theory*, Amsterdam: Elsevier, pp. 449-471.
- Putnam, H., 1967. ‘Mathematics without Foundations’, in Benacerraf & Putnam 1983, 295-311.
- Putnam, H., 1972. *Philosophy of Logic*, London: George Allen & Unwin.
- Quine, W.V.O., 1969. ‘Epistemology Naturalized’, in W.V.O. Quine, *Ontological Relativity and Other Essays*, New York: Columbia University Press, pp. 69-90.
- Quine, W.V.O., 1970. *Philosophy of Logic*, Cambridge, MA: Harvard University Press, 2nd edition.
- Reck, E. & Price, P., 2000. ‘Structures and Structuralism in Contemporary Philosophy of Mathematics’, *Synthese*, 125: 341-383.
- Resnik, M., 1997. *Mathematics as a Science of Patterns*, Oxford: Clarendon Press.
- Russell, B., 1902. ‘Letter to Frege’, in van Heijenoort 1967, 124-125.

- Shapiro, S., 1983. ‘Conservativeness and Incompleteness&rsquop;, *Journal of Philosophy*, 80: 521-531.
- Shapiro, S., 1991. *Foundations without Foundationalism: A Case for Second-order Logic*, Oxford: Clarendon Press.
- Shapiro, S., 1997. *Philosophy of Mathematics: Structure and Ontology*, Oxford: Oxford University Press.
- Sieg, W., 1994. ‘Mechanical Procedures and Mathematical Experience’, in A. George (ed.), *Mathematics and Mind*, Oxford: Oxford University Press.
- Tait, W., 1981. ‘Finitism’, reprinted in Tait 2005, 21-42.
- Tait, W., 2005. *The Provenance of Pure Reason: Essays in the Philosophy of Mathematics and its History*, Oxford: Oxford University Press.
- Troelstra, A. & van Dalen, D., 1988. *Constructivism in Mathematics: An Introduction* (Volumes I and II), Amsterdam: North-Holland.
- Turing,; A., 1936. ‘On Computable Numbers, with an Application to the Entscheidungsproblem’, reprinted in M. Davis (ed.), *The Undecidable: Basic Papers on Undecidable Propositions and Uncomputable Functions*, Hewlett: Raven Press, 1965, pp. 116-151.
- Tymoczko, T., 1979. ‘The Four-Color Problem and its Philosophical Significance’, *Journal of Philosophy*, 76: 57-83.
- van Atten, M., 2004. *On Brouwer*, London: Wadsworth.
- van Heijenoort, J., 1967. *From Frege to Gödel: A Source Book in Mathematical Logic (1879-1931)*, Cambridge, MA: Harvard University Press.
- Weir, A., 2003. ‘Neo-Fregeanism: An Embarrassment of Richess’, *Notre Dame Journal of Formal Logic*, 44: 13-48.
- Weyl, H., 1918. *The Continuum: A Critical Examination of the Foundation of Analysis*, Stephen Pollard and Thomas Bole (trans.), Mineola: Dover, 1994.
- Woodin, H., 2001a. ‘The Continuum Hypothesis. Part I’, *Notices of the American Mathematical Society*, 48: 567-578.
- Woodin, H., 2001b. ‘The Continuum Hypothesis. Part II’, *Notices of the American Mathematical Society*, 48: 681-690.
- Wright, C., 1983. *Frege's Conception of Numbers as Objects*, (Scots Philosophical Monographs, Volume 2), Aberdeen: Aberdeen University Press.
- Zach, R., 2006. ‘Hilbert's Program Then and Now’, in D. Jacquette (ed.), *Philosophy of Logic*, (Handbook of the Philosophy of Science, Volume 5), Amsterdam: Elsevier, pp. 411-447.

# The End
# Thank you

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Questions?
Comments?
Discussion?

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