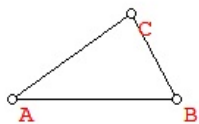


1. Quadrature Problem: Given a region in the plane, find a root so that the square of this root has the same area.

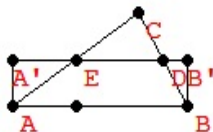
a. Quadrature of a Triangle: Given  $\triangle ABC$ ,

Find a square  $\square DEFG$  with root = DE and area of  $\square DEFG = \text{area of } \triangle ABC$

i. Construct a rectangle equal in area to that of  $\triangle ABC$



ii. Construct a square equal in area to the rectangle.



b. Quadrature for Polygons:

Problem: Find the root of a square that has the same area as a given polygon.

Suggest the outline for a procedure to accomplish the solution of the problem..

Hint: Use triangles and the Pythagorean Theorem.

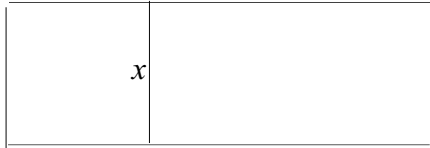
2. Example for completing the square problem:

[ al'Khowarizmi  $\approx$  820 AD and al'Khayyam  $\approx$  1100 AD.]

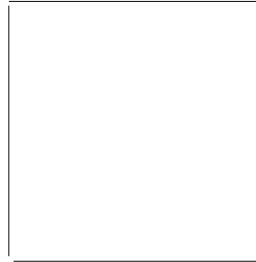
Find the root of the square which when added to a rectangle with one side of the same length as the root gives a rectangle of area  $c$ .

$x$

$b$



has area  $c$ :



$$x^2 + bx = c, \text{ has root } x = \sqrt{(b/2)^2 + c} - b/2$$

3. Descartes Arithmetic for Segments:

a. Multiplication using a unit segment and proportional sides of similar triangles.

b. Square roots using a unit segment and right triangles in a semicircle.

4. Descartes Arithmetic for Line Segments;

a. Multiplication

b. Square Roots

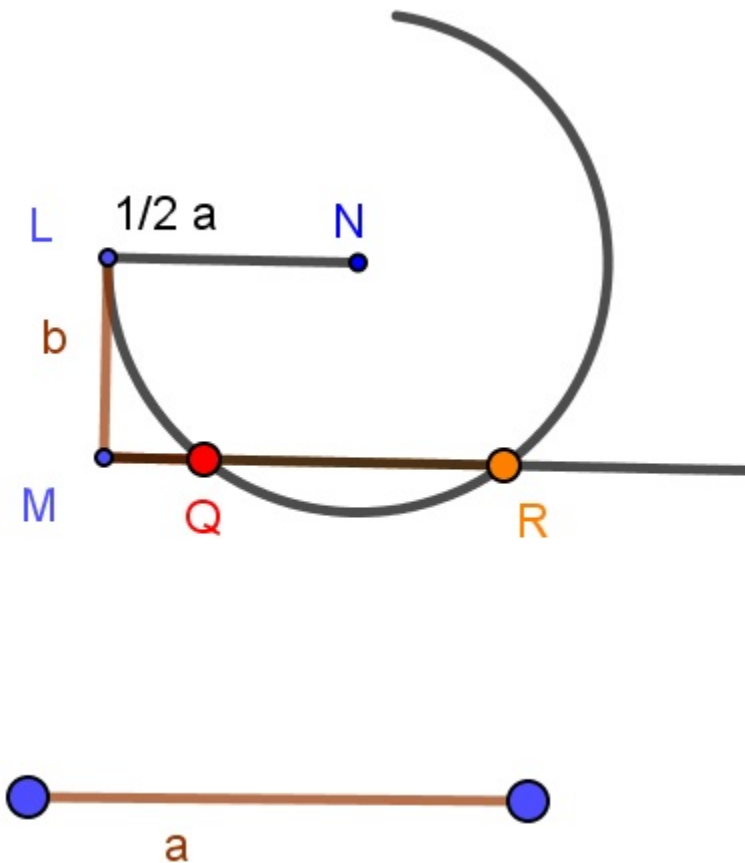
5. Descartes solves a quadratic equation for the arithmetic of segments.

$$z^2 = az - b^2$$

NL =  $1/2 a$ , LM =  $b$ ,  $NL \perp LM$ . MQR || LN

Circle with center N, through L, meeting MQR at Q and R.

Show that MQ and MR are solutions for  $z$  in the equation. [Hint: Use the Pythagorean Theorem]



6. Suppose  $f(x) = x^2 - 4x + 2$

a. Draw a sketch of the graph  $g(x) = f(x) - 2$  by finding the roots of  $g$ .

b. Find the axis of symmetry for  $g$  and  $f$ .

c. Express  $f$  in the vertex form (“completing the square”).

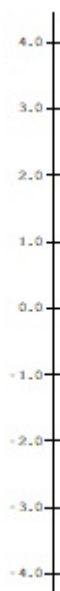
d. Solve the equation:  $f(x) = x^2 - 4x + 2 = 0$ .

7.

- a. Complete the following tables for  $m(x) = 2x$  and  $s(x) = x + 1$

$x$	$m(x) = 2x$	$s(x) = x + 1$
2		
1		
0		
-1		
-2		

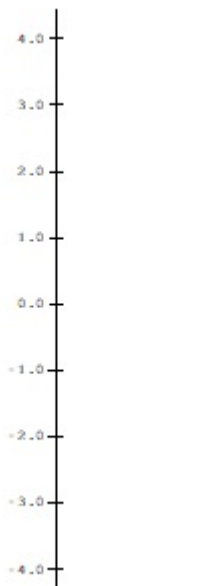
- b. Using the data from part a), on separate diagrams sketch mapping diagrams for  $m(x) = 2x$  and  $s(x) = x + 1$



8. Let  $q(x) = x^2$ .  
a. Complete the following table for  $q(x) = x^2$ .

$x$	$q(x) = x^2$
2	
1	
0	
-1	
-2	

- b. Using the data from part a), sketch a mapping diagram for  $q(x) = x^2$ .



9. Solving  $2(x-3)^2 + 1 = 9$  with a mapping diagram.

a. Express  $f(x) = 2(x-3)^2 + 1$  as composition of core linear and quadratic functions.

$$f(x) = h(m(q(z(x)))) \text{ where}$$

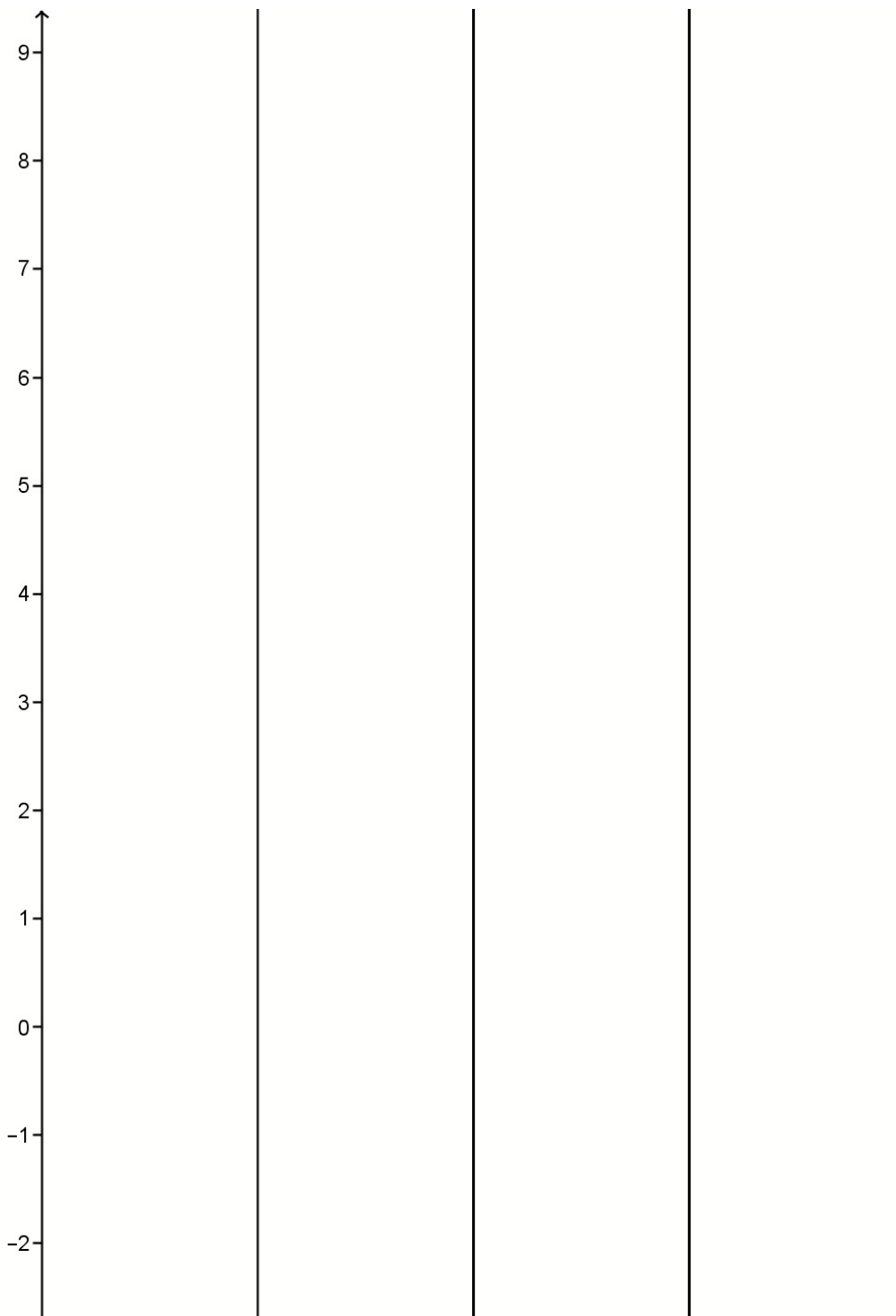
$$h(x) = \underline{\hspace{2cm}}$$

$$m(x) = \underline{\hspace{2cm}}$$

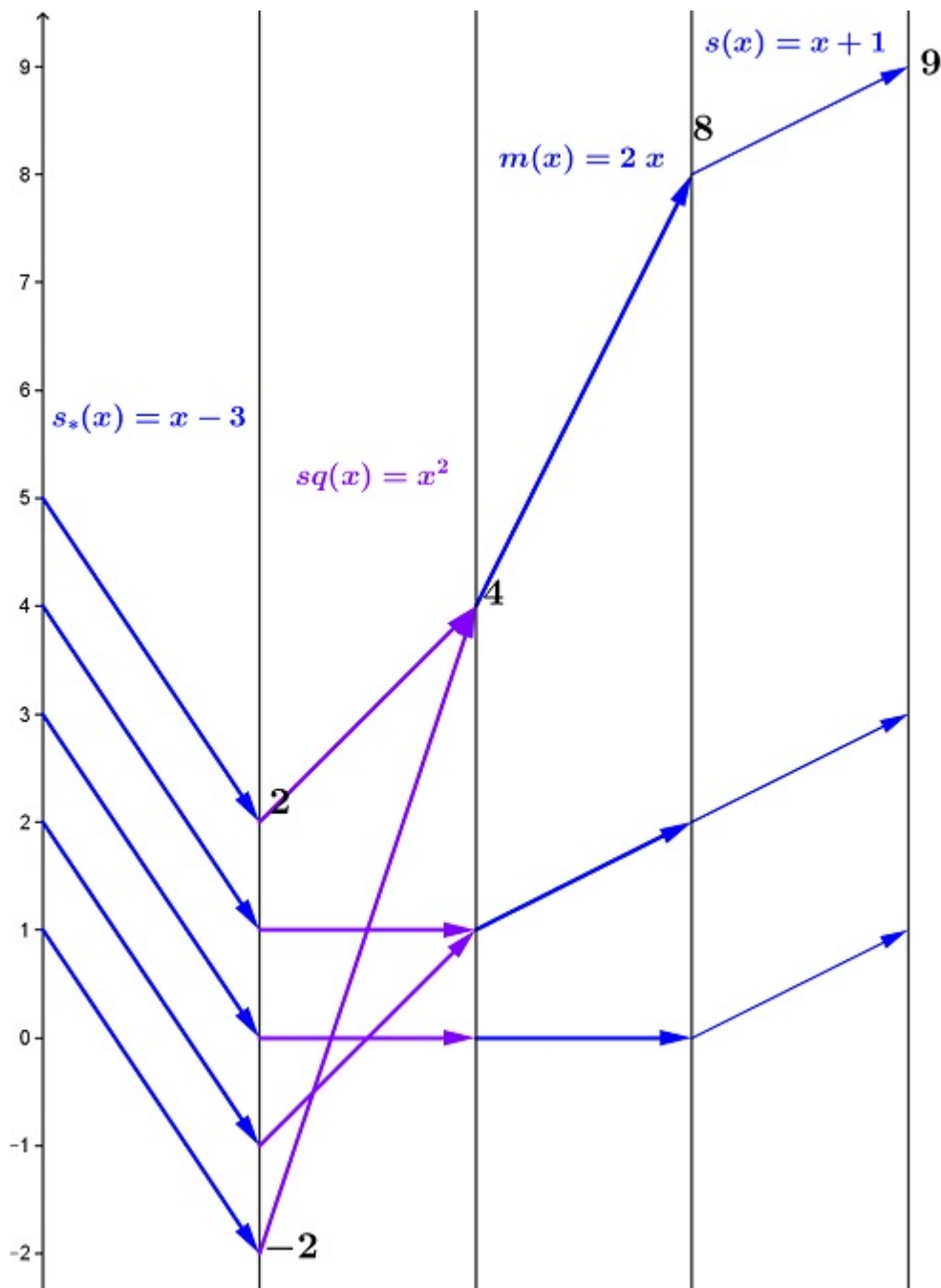
$$q(x) = \underline{\hspace{2cm}}$$

$$z(x) = \underline{\hspace{2cm}}$$

b. Sketch a mapping diagram for  $f$  as a composition.



- c. On the mapping diagram below indicate by circling numbers and arrows how the diagram visualizes the solution of  $2(x-3)^2 + 1 = 9$ . **Check the solutions.**



Check: