Is Philosophy of Mathematics Important for Teachers?

Preliminary report.

JMM 2016 - POMSIGMAA CP Session

Using Philosophy to Teach Mathematics

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Abstract

There has been much interest in recent years on what mathematical preparation is important for future teachers at all levels.

Recommendations from the MAA CUPM on Undergraduate Curriculum and the Common Core in Mathematics are silent on the issue of what role the philosophy of mathematics can play.

The author will suggest examples where a discussion of some issues from the philosophy of mathematics in courses taken by future teachers can enrich their backgrounds and training.

What is Mathematics? B. Russell

Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.

– **Bertrand Russell**, *Mysticism and Logic (1917) ch. 4*

What is Mathematics? B. Russell

Mathematics may be defined as the subject in which we never know what we are talking about

[Ontology: POM], [HUMILITY? Not POM!]

nor whether what we are saying is true [Epistemology: POM].

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– **Bertrand Russell**, *Mysticism and Logic (1917) ch. 4*

What is Philosophy of Mathematics?

- **Ontology** for Mathematics: "Being"
- Ontology studies the nature of the "objects" of mathematics.
- Some issues for discussion in courses taken by future teachers:
  - What is a **number**?
  - What is a **point**? **line**?
  - What is a **set**?
  - In what sense **do these objects exist**?

What is Philosophy of Mathematics?

- Epistemology for Mathematics: "Knowing"
- Epistemology studies the acquisition of **knowledge of the truth** (of a mathematical statement).
- Some issues for discussion in courses taken by future teachers:
  - Does knowledge come from experience and evidence?
  - Does knowledge come from argument and proof?
  - Is knowledge relative or absolute?

What is Philosophy of Mathematics?

Sample of Some POM's
[with Flashman Interpretations]

- Platonism (P)
- Empiricism (E)
- Formalism (F)
- Constructivism (C)
- Structuralism (S)

What is Philosophy of Mathematics?

Platonism (P)

[Flashman Interpretation]

- Math objects are **postulated abstract entities underlying common experience**.
- Knowledge comes from logic and reasoning and a fundamental core, without verification through direct experience.

What is Philosophy of Mathematics?

Empiricism (E)

[Flashman Interpretation]

- Math objects are part [a quality?] of **physical reality.**
- Knowledge is connected to experience and evidence combined with logic and reasoning.

What is Philosophy of Mathematics?

Formalism (F)

[Flashman Interpretation]

- Math objects are constructed through the creation of **formal systems represented with symbolic objects** and manipulated with formal rules.
- Knowledge derives through formal rules of logic and reasoning by symbolic manipulation based on primitive formal postulates.

What is Philosophy of Mathematics?

Constructivism (C)

[Flashman Interpretation]

- Math objects are **constructed through human cognitive processes** (but are not subjective mental objects).
- Knowledge evolves through constructions using logic and reason based on primitive constructed knowledge.

What is Philosophy of Mathematics?

Structuralism (S)

[Flashman Interpretation]

- Mathematics studies **structures, theories and systems that reflect [common] relational concepts [not objects],** often useful in scientific study.
- Knowledge evolves through logic and reason relative to the structure/theory/system under investigation.

What Mathematical Preparation is Important for Future Teachers?

- The Common Core **Standards**
  - **By Domain [Flashman Summary]**
    - **Number**
    - **Algebra**
    - **Geometry**
    - **Expressions & Equations**
    - **Functions**
    - **Statistics & Probability**
  - By Mathematical Practices

Common Core
Standards by Domain

- **Counting & Cardinality ***
- Operations & Algebraic Thinking
- **Number & Operations in Base Ten ***
- **Number & Operations—Fractions ***
- **Measurement & Data**
- Geometry
- **Ratios & Proportional Relationships**
- **The Number System**
- Expressions & Equations
- Functions
- Statistics & Probability

Common Core

A Progressive Understanding of Number

- **Counting** ***[POM Issue & Emphasis added]**
  - Understand the relationship between numbers and quantities; **connect counting to cardinality**. **[Cardinality, C]**
  - **When counting objects**, **say the number names in the standard order**, **pairing** each object with one and only one number name and each number name with one and only one object. **[Ordinality, F]**
  - Understand that the **last number name** said tells the **number of objects counted**.
    The number of objects is the same regardless of their arrangement or the order in which they were counted. **[Abstraction, P]**
  - Understand that each **successive number name** refers to a **quantity** that is **one larger**. **[Ordinality, F, P, C]**

Common Core

A Progressive Understanding of Number

- **Number & Operations in Base Ten ***
  [POM Issue & Emphasis added]
  - Understand **place value**. **[F]**
  - **Generalize** place value understanding for multi-digit whole numbers. **[Abstraction? F,P]**
  - Understand the **place value system**. **[F, C, P, S]**

Common Core

A Progressive Understanding of Number

- **Number & Operations—Fractions ***
  [POM Issue & Emphasis added]
  - Develop understanding of fractions as numbers. **[S]**
  - Understand a fraction 1/*b* as the quantity formed by **1 part when a whole is partitioned into *b* equal parts**; understand a fraction *a*/*b* as **the quantity formed** by *a* parts of size 1/*b*. **[C]**
  - Understand a fraction as a number on the number line; represent fractions on a number line diagram.**[E, S]**
  - Explain equivalence of fractions in **special cases**, and compare fractions by **reasoning about their size**. **[E, Abstraction, P]**

Common Core

A Progressive Understanding of Number

- **Number & Operations—Fractions (CONT'D.)**
  [POM Issue & Emphasis added]
  - **Extend** understanding of fraction equivalence and ordering. **[C, Abstraction, P]**
  - **Build** fractions from unit fractions. **[C]**
  - Understand **decimal notation** for fractions, and compare decimal fractions. **[F, S]**
  - **Apply and extend** previous understandings of multiplication and division. **[C, S, Abstraction, P]**

-Common Core-

A Progressive Understanding of Number

Exercise?: Identify POM Issues

- **<u>Ratios</u> & Proportional Relationships**
  - Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
  - Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship.

-Common Core-

A Progressive Understanding of Number

Exercise?: Identify POM Issues

- **The Number System**
  - **Apply and extend previous understandings of numbers to the system of rational numbers.**
  - Understand that positive and negative numbers are used together to describe quantities having opposite directions or values.
  - Understand a rational number as a point on the number line.

-Common Core-

A Progressive Understanding of Number

Exercise?: Identify POM Issues

- **The Number System (cont'd.)**
  - Understand ordering and absolute value of rational numbers.
  - Understand the absolute value of a rational number as its distance from 0 on the number line.
  - **Know that there are numbers that are not rational, and approximate them by rational numbers.**
  - Understand informally that every number has a decimal expansion.

-Common Core-
Mathematical Practices

Exercise?: Identify POM Issues

- Make sense of problems and persevere in solving them.
- **Reason abstractly and quantitatively.**
- **Construct viable arguments** and critique the reasoning of others.
- **Model with mathematics.**
- **Use appropriate tools strategically.**
- Attend to precision.
- **Look for and make use of structure.**
- **Look for and express regularity in repeated reasoning.**

-Common Core-

Mathematics Glossary Excerpts

- **Fraction.** A number **expressible in the form a/b** where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) *See also:* rational number.
- **Integer.** A number **expressible in the form a or -a** for some whole number a
- **Number line diagram.** A diagram of the number line used to **represent numbers support reasoning about them**. In a number line diagram for measurement quantit the interval from 0 to 1 on the diagram represents the **unit of measure** for the quantity.
- **Rational number.** A number **expressible in the form a/b or - a/b** for some fraction a/b. The rational numbers include the integers.
- **Repeating decimal.** The decimal **form** of a rational number. *See also:* terminating decimal.
- **Terminating decimal.** A decimal is called terminating if its repeating digit is 0.
- **Whole numbers.** The numbers **0, 1, 2, 3, ...**

Observations on Teaching and Learning (with Common Core)

- Student Learning Outcomes are usually framed as
  - capabilities for actions or
  - achieving understanding indicated by related capabilities.
- There are multiple approaches to concept development that are based on different philosophical models for understanding.

Importance for POM in Teaching and Learning

- For the Teacher/Mentor (T/M)
  - Awareness of philosophical issues can alert the T/M to excessively authoritarian approaches.
  - Alternative philosophical views can allow the T/M to use and/or develop alternatives to traditional approaches.
  - Philosophical issues can illuminate the value of and need for developing a variety of mathematical tools for "solving problems."

Importance for POM in Teaching and Learning

- For the Student/Learner (S/L)
  - Helps the S/L understand the context, goals, and objectives of the mathematics being studied.
  - Alerts the S/L to the use of authority and the value and utility of different approaches to mathematics.
  - Opens the S/L to considerations of the human values and assumptions made in developing and using mathematics.

Exploring Example(s)

Following are examples of topics that can be used to introduce and explore some philosophical issues for teachers at a variety of levels.

- Consider how these examples can be expanded or transformed to explore other teaching situations and connections to the philosophy of mathematics.
- Consider how these examples can be expanded or transformed to other mathematics topics at various levels of school instruction.

Is Three An Odd Number?

- Questions for Open Discussion in a Course for Teachers
- **Ontological:**
  - Definitions?
  - What is three? [Does it exist? Is it one thing?]
    - Is three : 3, s(2), s(s(s(0))), {∅,{∅},{∅,{∅}}}, {{{∅}}}, #{a,b,c}?
    - Is 2 bigger then 3 ?
    - Is 2 ∈ 3?
  - What is the nature of this "object"?

Is Three An Odd Number?

- Questions for Open Discussion in a Course for Teachers
- **Epistemological**
  - How do we know three exists?
    - Different responses based on philosophy: P, F, E, C, S,
  - What defines "odd" number?
  - How do we know two is not an "odd" number?
    - Different responses based on philosophy: P, F, E, C, S?
  - How do we show three satisfies this definition?
    - Different responses based on philosophy: P, F, E, C, S?

-The Square Root of Two-

- Questions for Open Discussion
- Ontological:
  - Definition?
  - Does it exist?
  - What is the nature of this object?
- Epistemological
  - How do we know it exists?
  - How do we know it is "between 1 and 2"
  - How do we know it is not a rational number?

-The Square Root of -1: "*i*"-

- Questions for Open Discussion
- Ontological:
  - Definition?
  - Does it exist?
  - What is the nature of this object?
- Epistemological
  - How do we know ***i*** exists?
  - How do we know ***i*** is not a real number?

Is Philosophy of Mathematics Important for Teachers?

VOTE ?!

Yes?.....

No? .....

What to do? Two Suggestions +

- Organize workshops for mathematics education faculty and teachers (pre- and in-service) introducing the issues of philosophy of mathematics and their relevance to the school (Common Core) curriculum.
- Organize conferences/communications for developing teaching materials and resources that involve mathematics education faculty and teachers at a variety of levels in the issues of philosophy of mathematics.
- Funding? NSF, NEH, Corporate and Foundation

MAA Session on Common Core State Standards (CCSS) for Mathematics Practices and Content

The Role of Math Departments in Preparing Math Education Candidates for New Assessments

Thursday January 7, 2016

1:00 p.m.-4:15 p.m.

Room 303, Washington State Convention Center

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The End

Questions?

Comments?

Discussion?

This Presentation:

http://users.humboldt.edu/flashman/Presentations/JMM2016/POM4Teachers.pdf