CONCEPTS TO DRIVE TECHNOLOGY

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Introduction: The interaction of **form and substance** in the development of mathematics has a long and remarkable history. In the last decade of this century we can expect this interaction to continue bringing new results and approaches to the many problems transferred to mathematics from other disciplines. One relatively new aspect of this interaction is the application of technology for both instruction and research in mathematics.

Technology allows us to manage many more individual pieces of information than ever before. We can organize our knowledge with much greater ease and present it with even greater varieties of expression. Why just the number of fonts available to secretaries and deans in writing memos on their computer/word processors can make one's head spin.

In mathematics reform, especially in the pre-calculus and calculus curricula, technology (either hand-held, lap-top, desktop, or floor-top) has been an instrument for change in many ways. It has been used to alleviate the tedium of repetition, to accelerate the visualization of relations, to compute rapidly and efficiently, and to manipulate the symbols we have defined to represent problems and their solutions. This technology is heralded as bringing new power to its users, student and scholar alike, with its increased ability to interact as well in a wide variety of forms.

One has only to listen a short time to almost any of the speakers of the current calculus reform movement to hear tell of this power for the users of technology. Students will now take more control, become more active learners, explore and investigate mathematics more independently, and all because of technology. It's ways are ways of cooperation, and it's paths lead to truth and understanding. It opens the classroom to a new spirit and relation between student and mentor and makes real problems the subject matter.

Despite all these good things that technology brings we must be wary of some subtle dangers of this technology that is praised by so many. All of this technology brings with it a form, designed by its creators, by which we interact with it. And in this form the creators can control more or less the results of our investigations, the substance that we wish to comprehend.

The forms that we find in the design of technology and our interaction with it may limit the outcome of our supposed new found technological power. The design of technology is still limited by older views and forms of how to organize mathematical information. In becoming comfortable with technology we as teachers and users must be aware of the limitations and opportunities that technology presents. There are still many who hold a conservative view that no one can use a computer who cannot write a program in some high / low level computer

language such as BASIC, FORTRAN, or PASCAL. These views are certainly unhelpful to initiating students to technology. And in fact a student who is proficient in programming without having used the computer to explore and develop an approach to problem solving with the computer is perhaps handicapped further in working on a a more challenging. Time that would otherwise be used to creative though may be wasted in plodding through the drudgery of meaningless code organized in dull routines that take too long to find out useless information. Yet the more liberal view that favors menu and choice driven systems that never allow a user to define a new procedure are almost equally unhelpful. The limitations of pre-fabricated forms (menus, *et al.*) upon concept formation or even on the interaction between established concepts and their applications can be as severe as the overemphasis on syntax and programming.

Robust Concept Adaptability: In this paper I will discuss some examples of concept based technology applications. I suggest here that the technological power of the tools we provide our students can be judged by how they can address these (and other) concept driven applications blending form and substance. We can then distinguish between levels of concept capable technology. These distinctions are not based on the speed that the technology implements the same algorithm, or the ease with which a particular algorithm can be implemented or transformed to a useable version in a software or hardware environment. The added consideration is how well suited the technology is to adapt to newer concepts, for which the technology was not predesigned. Of course in the educational transitions to greater power, it may be convenient for the technology to include some of these concept applications as primitive features. However, the full measure for the power of technology needs to take into account the robust quality of concept adaptability.

The Rule of Three: It is almost obligatory when discussing technology in reform today to mention some form of the Rule of Three. That is, the rule that states that **when possible mathematical concepts should be presented using 1) numerical computation, 2)graphic visualization, and 3) algebraic/symbolic representation.** This rule is useful and seems to be justified historically by the triumph of many mathematical theories that have been built by promoting this three fold approach. The rule is however not without danger as a formal tool for instruction. One may be fooled by this rule. To prevent errors in the application of the rule of three, I wish to articulute

Rule 0: The application of the rule of three does not guarantee that instruction based on this rule will succeed. The rule works best when it is applied to some appropriate content message, a mathematical concept. Let me therefore make explicit a premiss, Rule 0, as a prerequisite for any application of the Rule of Three.

Rule 0: The Rule of Content and Sense

Before proceeding to develop mathematics using the Rule of Three, have something to say. Try to have what you say make sense both by itself and with the other parts of mathematics.

Formal application of the Rule of Three can give the appearance of reform, but without a change a sensible message, the form may deliver only a hollow message of little or no value to the student.

The Elements of Technological Power: What are the key elements of technology that students should have within their grasp for working on mathematics? The following list some key elements in describing a plausible ordering of the levels of control:

0. Arithmetic, core functions, evaluation.
1. a. User defined functions, tables, matrices, graphs.
   b. Statistical operations.
2. Interaction of data....(The Rule of Three)
3. Core mathematical operations.
   Differentiation / Integration / Sums
4. Logical Control and Programming

In choosing technology for instruction one needs to balance the ease of its use and its learning curve response time with the technology's power for robust concept adaptability. Though slide shows, hypercard applications, and menu driven software are all appropriate for many situations, students should have access eventually to some software at the fourth level of control.

As with other types of literacy, the development of technological literacy happens in stages. Initiation to technology may be made gradually with work requiring only level 0 or 1 control, planning for progressive elevations in the application of the available power as the user becomes more comfortable with the basic operation of the technology. Materials (such as Notebooks in Mathematica and model files in other systems) designed for more powerful technology at level 2 and 3 can operate like menus to prepare the student for later taking full control. These stages of development may or may not take place on the same technology. What is important is that we realize that to foster this development we need to expose our students to powerful technology in a way that encourages them to take more control themselves. With more control of technology, the user has greater ability to drive the technology with concepts rather than being limited by the form of application implicit in the technology's design.

Let's look at an example of a concept that has made such an impact on the design of technology that I have included it as an element of technology at the first level of power. [For this discussion I will be using *X(PLORE)*, [1] software developed by David Meredith of San Francisco State University, previously known as The Calculus Calculator. Though it lacks some features like symbolic solution of integrals, it would do well in a full evaluation by my standards. (*I.e.*, I like it.)]

Visualizing Functions: It goes without saying today that mathematical technology today should be able to graph a function with ease. This means in fact that it can accept the definition of a function, evaluate the function, and make a visual correspondence through the coordinate plane between the arguments of functions and the corresponding values. The

visualization of these correspondences as points in the plane is a mathematical concept (perhaps first applied by Oresme in about 1350) that we often take for granted.

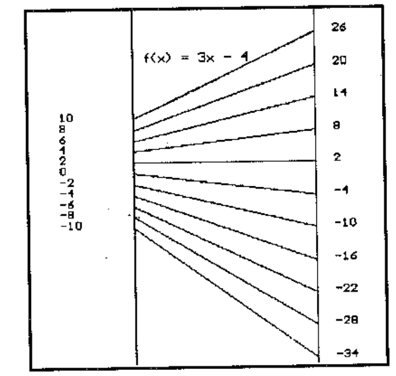

Figure 1
Transformation Figure for
f(x) = 3x - 4.

Let me suggest another conceptual visualization for a function and then discuss how this can drive the technology. The visual concept is what I call a **transformation (or mapping) figure.** Simply stated this figure shows the function correspondence by having two number lines, one for the arguments, (*i.e.*, numbers in the domain) called the source line, and a second for the values, (*i.e.*, numbers in range) called the target line. The correspondence is visualized by drawing a segment or arrow between the argument in the source line and its corresponding value determined by the function. This is not a difficult or particularly new concept for visualizing functions, but one that is not given much treatment in most current texts. [My own [6] excluded of course.] See Figure 1.

The issue is this: Can the technology we give our students implement this concept? and how easily can it adapt this concept for applications?

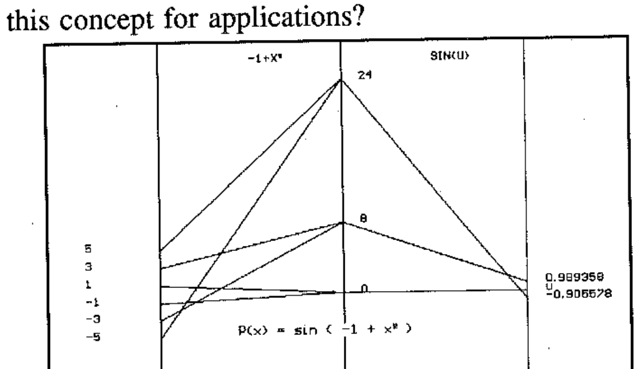

Figure 2
Transformation Figure for
a Composite Function

This concept is not difficult to implement in most technology environments that allow for some graphic programming. These range from graphing calculators like the Casio 7000 or the

TI 81 to the Colossus of Wolfram, Mathematica. Once some initial work is done with this concept it also becomes fairly easy to expand it's application to visualize the composition of functions [See Figure 2] as well as such basic notions as increasing, decreasing, and 1:1 functions. To the extent that the technology you and your students use cannot adapt to this concept, it restricts a students' ability to use this concept to its full advantage.

Tangent Fields, Integral Curves, and Indefinite Integrals. Moving to a slightly more advanced topic in the calculus curriculum, I will again focus on a mathematical concept used to visualize the relation given by a differential equation. When considering elementary ordinary differential equations in the first year of calculus, many writers now use the concepts of tangent (or direction) fields and integral curves, an approach that I have discussed for over 10 years. (For some recent discussion of this see [2], [3], and [5].)

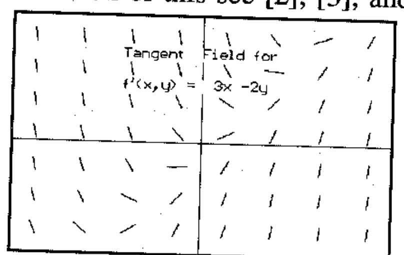

Figure 3
Tangent Field for
f'(x,y) = 3x -2y

Briefly, a differential equation can be considered to give an algebraic relation between the coordinates of a point in the plane and the slope of a line that would be tangent to a curve passing through that point that would be the graph of a solution to that equation. (That's complicated, but the picture will help make sense of the meaning here.) See Figure 3. A sketch showing selected segments of tangent lines through some points in the plane that satisfy the differential equation is called a **field of tangents or a tangent (direction) field.**

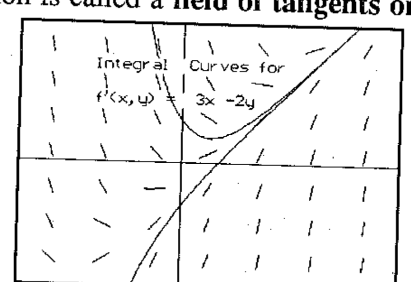

Figure 4
Integral Curves
for f'(x,y)= 3x - 2y

A curve that actually graphs a solution for the differential equation is called an **integral curve** for the equation. See Figure 4. These two concepts are available with many technology packages currently available for calculus and can be created without too much trouble in an environment that has some programming with graphics. Again the range of

technology that can produce these concepts is wide. But suppose we go further and wish to actually use these concepts in a situation where we can solve the differential equation by some other means. For the systems that have predesigned these concepts in a menu environment there is little opportunity to compare solutions concepts unless the designer of the technology foresaw the connection.

To be more specific, suppose the differential equation is

$$f'(x,y) = y \quad \text{or} \quad f'(x,y) = 1/(x^2 + 1) .$$

Would we be able to plot the exponential or arctangent functions that solve these differential equations on the associated tangent fields and compare them with integral curves drawn by some other means? And what if the equation is

$$f'(x,y) = 1/(x^2 + \cos^2(x))?$$

Then we should be able to compare the tangent field-integral curve solution to the graph of an indefinite integral defined by

$$f(x)=\int_0^x \frac{1}{t^2+\cos^2(t)}dt.$$

Probability and Calculus. As a last example let me turn to some of my recent efforts to bring probability applications into the calculus course in the study of continuous distribution functions. The concept here is that the derivative of the distribution function is the density function. To bring this out more concretely with technology I like to envision a unit circular magnetic dart board.

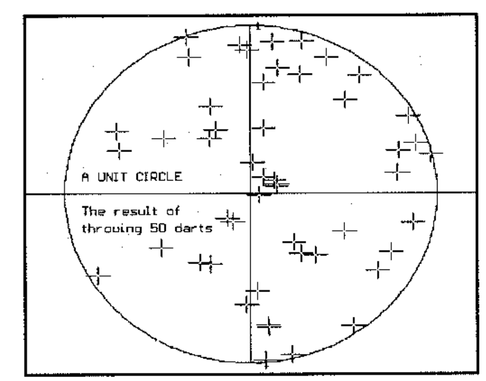

Figure 5
50 Darts on a
Unit Circle

Throwing darts at this board is simulated using a random number generator and some graphing commands in a program. [See Figure 5.] I know of no easy way to do this without programming capability. Data from this simulation can then be used interactively with statistical, algebraic, and calculus features to illustrate the notion of average probability density of a random variable for an interval. Modeling questions can then motivate the definition of the (point) probability density function as a limit, bringing the student to the concept of derivative with an interpretation that is far removed from tangents and rates. This application to random variables is more relevant to the actual needs of most science students in what they will deal with for most of their future experiences with mathematics. Once again a wide range of technology can achieve this interaction, but for some the task will be much easier to accomplish than others.

Conclusion: With these few examples I hope I have convinced you that there are mathematical concepts that can distinguish the level of power that technology brings to problem solving. We should not presume that we can start our students with technology at these higher levels of power. Our objective however should be to provide opportunities for our students to see this kind of power and to achieve it progressively as they develop both mathematical and technology literacy.

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SOME GENDER DIFFERENCES IN ATTITUDES AND MATHEMATICS PERFORMANCE WITH GRAPHICS CALCULATORS

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Introduction

The development of powerful and relatively cheap hand held graphics calculators has made it possible for graphing technology to be an integral part of the process of teaching and learning mathematics. However, while this brings with it many potential pedagogic benefits [1], we also need to be aware that we are bringing about a significant change in practice which will have a major impact on students, particularly those brought up in a tradition where mathematics has been viewed primarily as a paper and pencil activity. The impact of introducing graphing technology on students' attitude, performance and mathematical behaviour has been the focus of an on-going study at Swinburne University of Technology, where graphics calculators have been prescribed for first year calculus students since the beginning of the 1991 Australian academic year. This paper focuses on some findings related to gender differences in attitude and performance.

Educational Setting and Implementation Strategy

The calculus course involved some 350 students studying for a Bachelor of Applied Science. Approximately 35% of the students were female. Instruction consisted of 3 large group (100-150) lectures a week and 2 small group (25) tutorials/practice sessions. A typical US Calculus text was used [2]. Teaching staff were the faculty normally involved in the course. No changes were made to assessment procedures other than that students were allowed to use their graphics calculators in all assessment activities. No fundamental change was made to the material assessed. The calculator used was the TI-81.

Gender differences in attitude and performance

Student attitude to the introduction of the graphics calculator was assessed through large scale surveys done in both April (mid semester I) and September 1991 (mid semester II) (a copy of the survey instrument can be found in Boers & Jones, [3]). Final marks in a calculus course taken by all the students was used to assess student performance.

Attitude

When a gender break down of survey responses was made a number of differences in attitude were revealed [3]. The most important differences were that:

- in the first as well as second semester females rated themselves lower than males in their mathematical ability.
(χ^2 = 6.56, *df*=2, p=0.04 in April, and χ^2 = 5.03, *df*=2, *p*=0.08 in September);

- in the second semester there were more females than males who said they were anxious about mathematics
(χ^2 = 2.37, *df*=2, *p*=0.3 in April and χ^2 = 5.96, *df*=2, *p*=0.05 in September);

- in both semesters more females said that they found the TI-81 difficult to use
(χ^2 =18.40, *df*=2, *p*=0.0001 χ^2 =5.78, *df*=2, *p*=0.06);