Visualizing the Algebra of Equations with Mapping Diagrams

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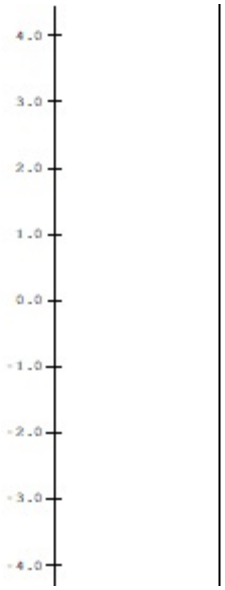
1.

a. Complete the following tables for $m(x) = 2x$ and $s(x) = x + 1$

x	$m(x) = 2x$	$s(x) = x + 1$
2		
1		
0		
-1		
-2		

b. Using the data from part a), on separate diagrams sketch mapping diagrams for $m(x) = 2x$ and $s(x) = x+1$



2. Let $q(x) = x^2$.

a. Complete the following table for $q(x) = x^2$.

x	$q(x) = x^2$
2	
1	
0	
-1	
-2	

b. Using the data from part a), sketch a mapping diagram for $q(x) = x^2$.

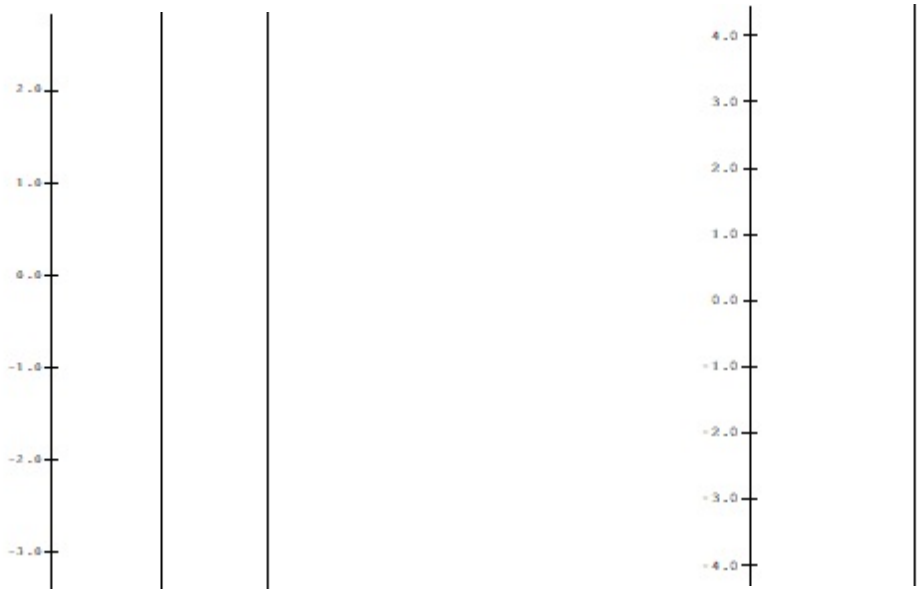
3.

a. Complete the following table for the composite function $f(x) = s(m(x)) = 2x + 1$.

x	$m(x) = 2x$	$s(m(x)) = 2\,x + 1$
2		
1		
0		
-1		
-2		

b. Use the table and the previous sketches of 1.b to draw a composite sketch of the mapping diagram with 3 axes for the composite function $f(x) = s(m(x)) = 2x + 1$.

c. Draw a sketch for the mapping diagram with 2 axes of $f(x) = 2\,x + 1$.

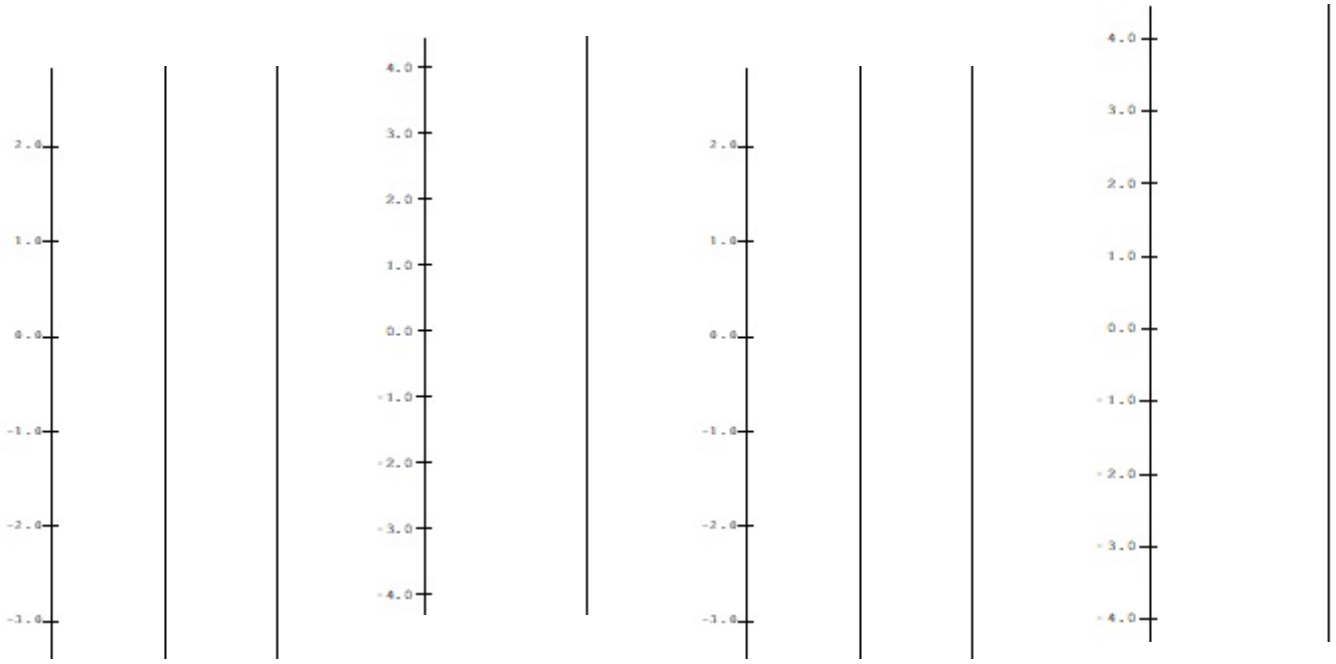


4. Let $q(x) = x^2$, $R(x) = s(q(x)) = x^2 + 1$, and $S(x) = q(s(x)) = (x+1)^2$.

   a. Complete the following tables for
   $q(x) = x^2$, $R(x) = s(q(x)) = x^2 + 1$ and $S(x) = q(s(x)) = (x+1)^2$

	$q(x) = x^2$	$R(x) = s(q(x)) = x^2 + 1$	$s(x) = x+1$	$S(x) = q(s(x)) = (x+1)^2$
2				
1				
0				
-1				
-2				

   b. Using the data from part a), on separate diagrams sketch mapping diagrams for the composition $R(x) = s(q(x)) = x^2 + 1$ and $S(x) = q(s(x)) = (x+1)^2$ with three axes and then two axes.



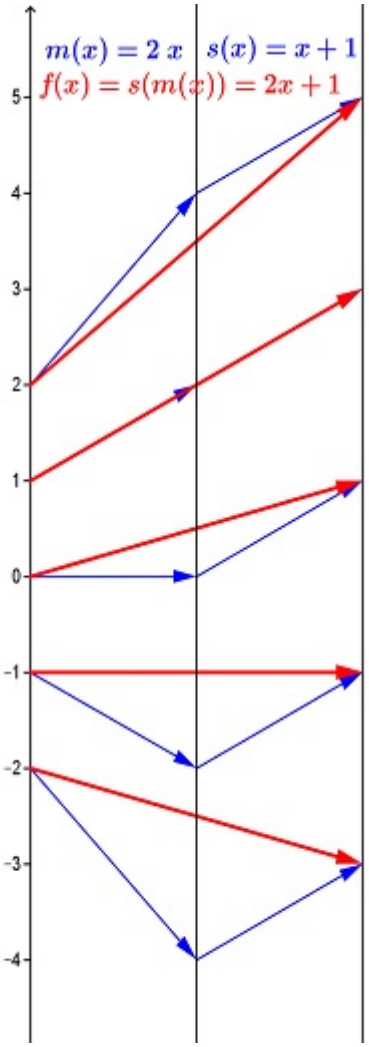
5. Solving Equations:
   a. Use a standard algebraic approach to solve the following equation. Show all steps. Check your answer.

$$2x + 1 = 5.$$

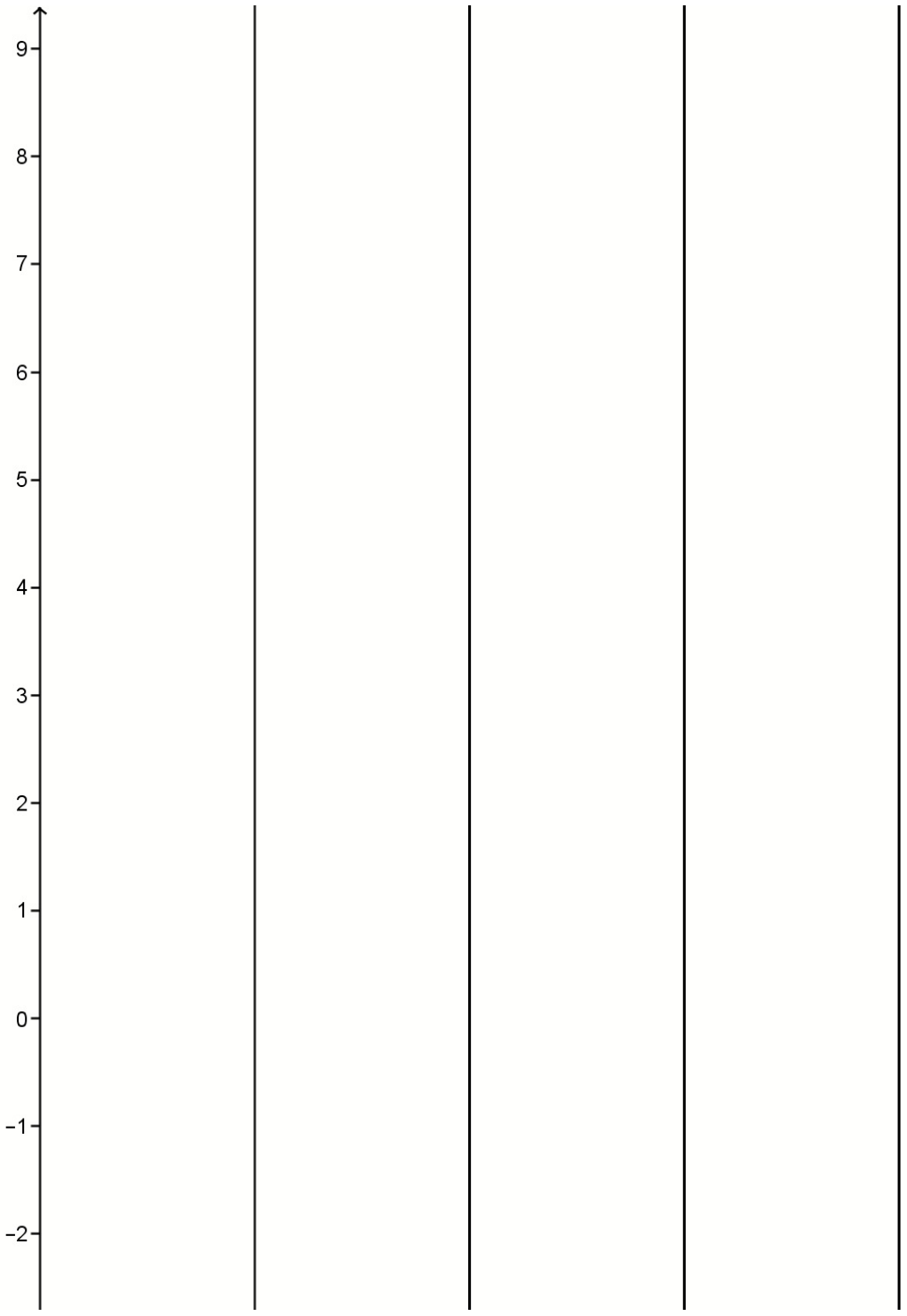
Work:

Check:

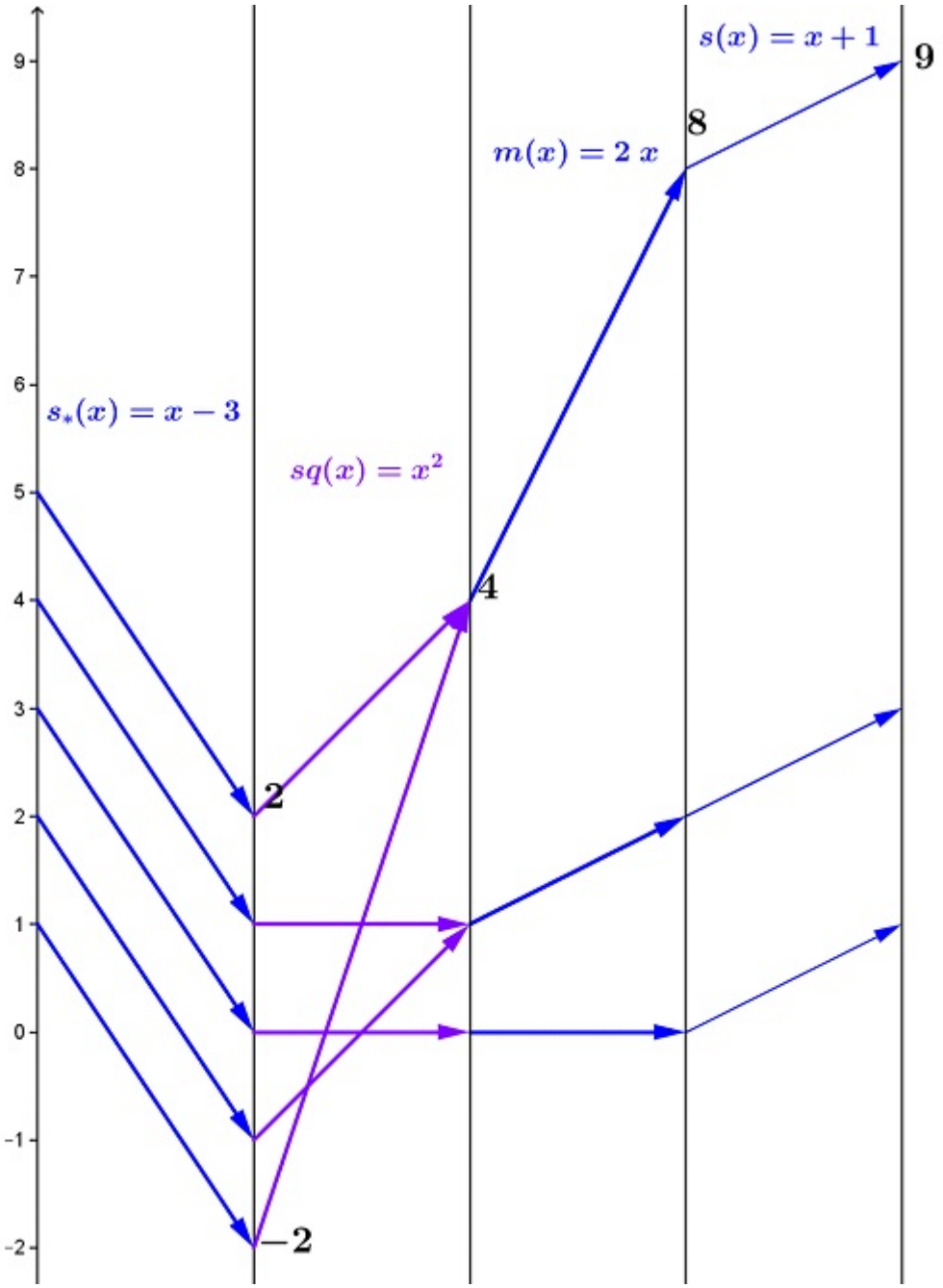
b. On the mapping diagram below indicate by adding and circling numbers and arrows how the diagram visualizes the work in your algebraic solution of $2x + 1 = 5$.



6. Solving $\mathbf{2(x-3)^2 + 1 = 9}$ with a mapping diagram.
   a. Express $\boldsymbol{f(x) = 2(x-3)^2 + 1}$ as composition of core linear and quadratic functions.
      $f(x) = h(m(q(z(x))))$ where
      $h(x) =$ _____
      $m(x) =$ _____
      $q(x) =$ _____
      $z(x) =$ _____
   b. Sketch a mapping diagram for f **as a composition**.



c. On the mapping diagram below indicate by circling numbers and arrows how the diagram visualizes the solution of **$2(x\text{-}3)^2 + 1 = 9$. Check the solutions.**

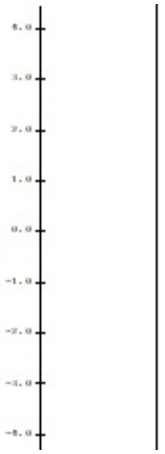


Check:

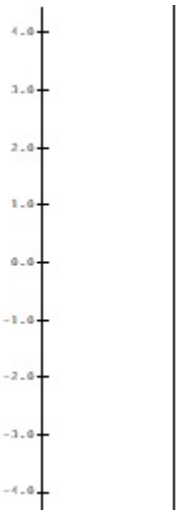
7. Let $f(x) = mx + b$. Sketch mapping diagrams for the following.
   Use the same scale for the second axis.

   a. m = -2; b = 1: $f(x) = -2x + 1$;

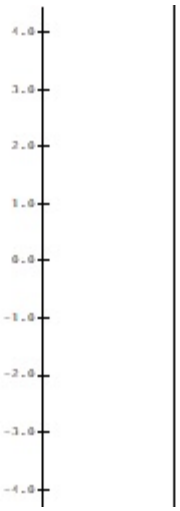
   b. m = 2 b = 1: $f(x) = 2x + 1$



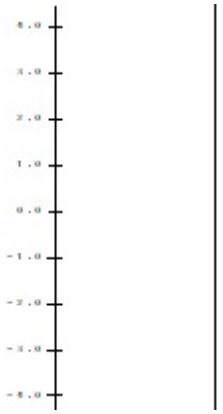
   c. m = ½; b = 1: $f(x) = ½ x + 1$



   d. m = 0; b = 1: $f(x) = 0 x + 1$



   e. m = 1; b = 1: $f(x) = x + 1$

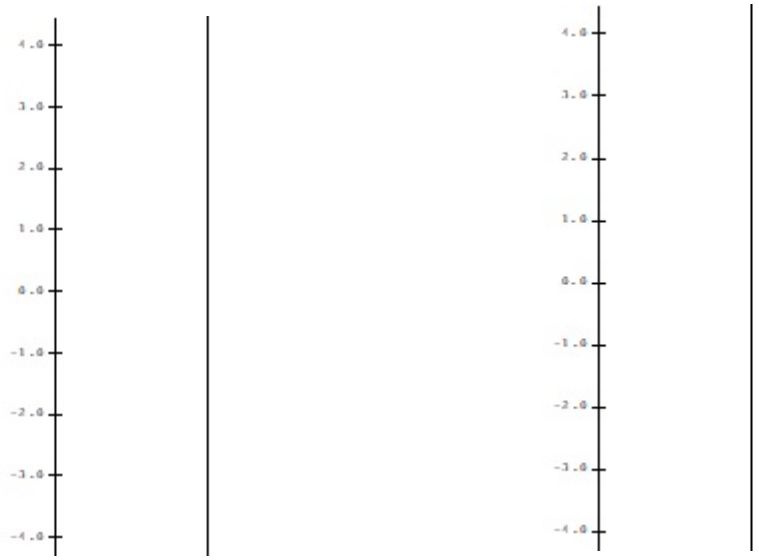


8. Exponential and logarithmic functions.

a. Complete the following table for $f(x) = 2^x$

x	$f(x) = 2^x$
2	
1	
0	
-1	
-2	

b. Using the data from part a), sketch a mapping diagram for $f(x) = 2^x$ and $g(x) = \log_2(x)$.



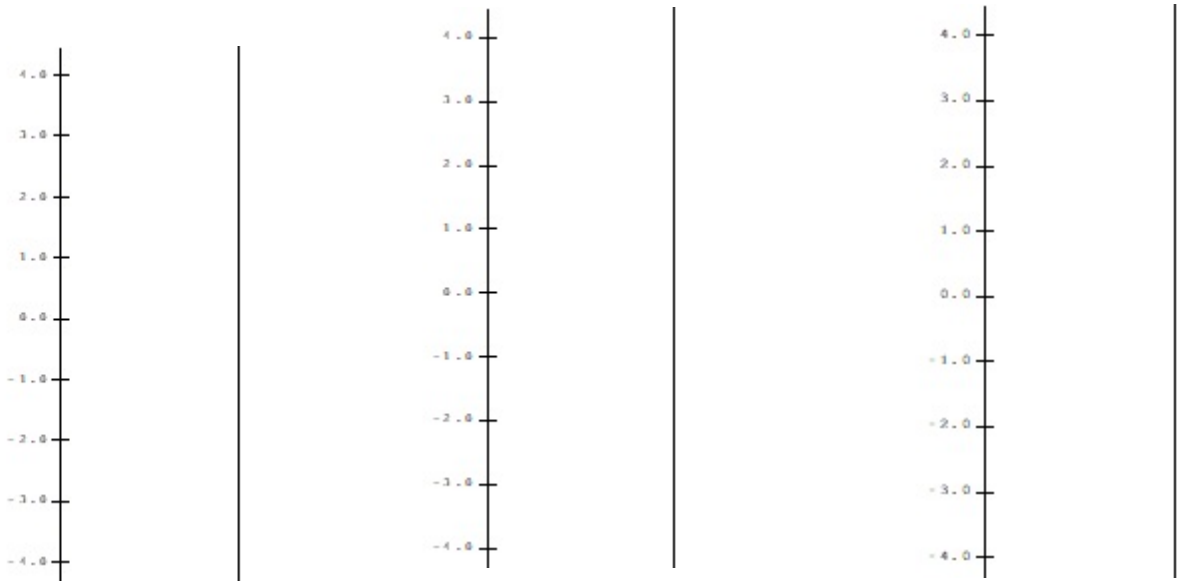
c. Discuss how the domain and range of f and g are evidenced in the mapping diagrams.

9. Core Trigonometric Functions

a. Complete the following table for the trigonometric functions sine, cosine, and tangent.

x	$\sin(x)$	$\cos(x)$	$\tan(x)$
π			
$3\pi/4$			
$\pi/2$			
$\pi/4$			
0			
$-\pi/4$			
$-\pi/2$			
$-3\pi/4$			
$-\pi$			

b. Change the scales on the following mapping diagrams and draw mapping diagrams for the trigonometric functions sine, cosine, and tangent.



c. Discuss how the periodic behavior of these functions will be evidenced in their mapping diagrams

Mapping Diagrams (2 and 3 Axes)