
1. Suppose A and B are finite sets. Prove: $\#(A \cup B) = \#(A) + \#(B) - \#(A \cap B)$.

[This was done in class.]

2. Suppose A, B, and C are finite sets.

Prove: $\#(A \cup B \cup C) = \#(A) + \#(B) + \#(C) - \#(A \cap B) - \#(A \cap C) - \#(C \cap B) + \#(A \cap B \cap C)$.

3. Using the binomial theorem:

a. Verify: $\binom{3}{0} - \binom{3}{1} + \binom{3}{2} - \binom{3}{3} = 0$

b. Verify: $\binom{3}{0}2^3 - \binom{3}{1}2^2 + \binom{3}{2}2 - \binom{3}{3} = 1$

c. Prove for any n : $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \dots + (-1)^n \binom{n}{n} = 0$

d. Prove for any n : $\binom{n}{0}2^n - \binom{n}{1}2^{n-1} + \dots + (-1)^{n-1} \binom{n}{n-1}2 + (-1)^n \binom{n}{n} = 1$

4. Suppose A and B are disjoint finite sets with $\#(A) = 5$ and $\#(B) = 3$. Find the number of ways to form an ordered 8-tuple so that no element of B is adjacent to another element of B and no elements appear twice. Prove that your result is correct.

5. Suppose A and B are disjoint finite sets with $\#(A) = n$ and $\#(B) = k$ with $k \neq n$. Give a formula for the number of ways to form an ordered $n + k$ -tuple so that no element of B is adjacent to another element of B and no elements appear twice. Prove that your formula is correct.

6. Look up “trinomial coefficients” on the internet. Give a definition for this term that connects these coefficients to counting sets. Relate these coefficients to the expansion of a power of a trinomial.