Math 240 Problem Set #12 M. Flashman
Spring, 2012

1. Suppose f: X → Y and g: Y → Z so $g \circ f$: X → Z. Prove the following:
   a. If f and g are onto (surjective) then $g \circ f$ is onto.
   b. If f and g are 1:1 (injective) then $g \circ f$ is 1:1.
   c. If f and g are bijective then $g \circ f$ is bijective.

2. Suppose f : X → Y. Prove the following:
   a. If $A \subseteq X$ then $A \subseteq f^{-1}(f(A))$. Give an example where $f^{-1}(f(A)) \neq A$.
   b. If $B \subseteq Y$ then $f(f^{-1}(B)) \subseteq B$. Give an example where $f(f^{-1}(B)) \neq B$.
   c. If b and c are elements of Y with $b \neq c$ then $f^{-1}(\{c\}) \cap f^{-1}(\{b\}) = \varnothing$ [revised 3/28]

3. Suppose f : X → Y and g : Y → X. Prove the following:
   a. If $g \circ f = id_X$ then f is 1:1 (injective).
   b. If $f \circ g = id_Y$ then f is onto (surjective).

4. Suppose f : X → Y and there is a function g: Y → X so that

i) $f \circ g = id_Y$ and ii) $g \circ f = id_X$.

Prove that the function g is <u>unique</u> with respect to functions satisfying properties i) and ii).

5. Suppose σ is a permutation of the set {1,2,3,4,5} and for each n, $\sigma \circ \sigma(n) = n$.
Prove that for some n, $\sigma(n) = n$. [Hint: Use an indirect proof.]
Bonus: Generalize this problem and prove your generalization.