Math 240 Problem Set #16 Counting M. Flashman
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1. Suppose A and B are finite sets. Prove that $\#(A\cup B) = \#(A) + \#(B) - \#(A\cap B)$.

2. Suppose A, B, and C are finite sets.

Prove: $\#(A\cup B\cup C) = \#(A) + \#(B) + \#(C) - \#(A\cap B) - \#(A\cap C) - \#(C\cap B) + \#(A\cap B\cap C)$.

3. Using the binomial theorem.

   a. Verify: $\binom{3}{0} - \binom{3}{1} + \binom{3}{2} - \binom{3}{3} = 0$

   b. Verify: $\binom{3}{0}2^3 - \binom{3}{1}2^2 + \binom{3}{2}2 - \binom{3}{3} = 1$

   c. Prove for any n: $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} + \ldots + (-1)^n\binom{n}{n} = 0$

   d. Prove for any n: $\binom{n}{0}2^n - \binom{n}{1}2^{n-1} + \ldots + (-1)^{n-1}\binom{n}{n-1}2 + (-1)^n\binom{n}{n} = 1$

4. Suppose A and B are disjoint finite sets with #(A) = 5 and #(B) = 3. Find the number of ways to form an ordered 8- tuple so that no element of B is adjacent to another element of B and no elements appear twice. Prove that your result is correct.

5. Suppose A and B are disjoint finite sets with #(A) = n and #(B) = k with $k \leq n$. Give a formula for the number of ways to form an ordered $n + k$ - tuple so that no element of B is adjacent to another element of B and no elements appear twice. Prove that your formula is correct.

6. Look up "trinomial coefficients" on the internet. Give a definition for this term that connects these coefficients to counting sets. Relate these coefficients to the expansion of a power of a trinomial.