

1. Let $S = \mathbb{R}^2 = \{(x,y) : x,y \in \mathbb{R}\}$.

a. Let $L_c = \{(x,y) \in \mathbb{R}^2 : x+y = c\}$, and $\mathcal{F} = \{L_c : c \in \mathbb{R}\}$. Prove \mathcal{F} is a partition of S .

b. Suppose A , B and C be real numbers with $A^2 + B^2 \neq 0$.

Let $L(A,B,C) = \{(x,y) \in \mathbb{R}^2 : Ax + By = C\}$. Let $\mathcal{F}(A,B) = \{L(A,B,C) : C \in \mathbb{R}\}$.

Prove $\mathcal{F}(A,B)$ is a partition of S .

c. Let $A = S - \{(0,0)\}$.

For (a,b) in A , let $\langle a,b \rangle = \{(x,y) \in A : \text{for some real number } t, x = ta \text{ and } y = tb\}$.

Let $\mathcal{F} = \{\langle a,b \rangle : (a,b) \in A\}$. Prove \mathcal{F} is a partition of A .

2. Suppose $f: \mathbb{R} \rightarrow \mathbb{R}$ is an onto function. Let $\mathcal{F} = \{f^{-1}(\{a\}) : a \in \mathbb{R}\}$. Prove \mathcal{F} is a partition of \mathbb{R} .

3. Suppose $f: A \rightarrow B$ is an onto function. Let $\mathcal{F} = \{f^{-1}(\{b\}) : b \in B\}$. Prove \mathcal{F} is a partition of A .

4. DS Problem 2.22