Math 240 Problem Set #13 M. Flashman
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1. Suppose f: A → B and g: B → C so $g \circ f$: A → C. Prove the following:
   a. If f and g are onto (surjective) then $g \circ f$ is onto.
   b. If f and g are 1:1 (injective) then $g \circ f$ is 1:1.
   c. If f and g are bijective then $g \circ f$ is bijective.

2. Suppose f: A → B. Prove the following:
   a. If $X \subseteq A$ then $f^{-1}(f(X)) \subseteq X$. Give an example where $f^{-1}(f(X)) \neq X$.
   b. If $Y \subseteq B$ then $f(f^{-1}(Y)) \subseteq Y$. Give an example where $f(f^{-1}(Y)) \neq Y$.
   c. If b and c are elements of B with $b \neq c$ then $f^{-1}(\{a\}) \cap f^{-1}(\{b\}) = \varnothing$

3. Suppose f: A → B and g : B → A. Prove the following:
   a. If $g \circ f = id_A$ then f is 1:1 (injective).
   b. If $f \circ g = id_B$ then f is onto (surjective).

4. Suppose f: A → B and there is a function g: B → A so that

i) $f \circ g = id_B$ and ii) $g \circ f = id_A$.

Prove that the function g is unique with respect to functions satisfying properties i) and ii).

5. Suppose σ is a permutation of the set {1,2,3,4,5} and for each n, σ∘σ(n) = n.
Prove that for some n, σ(n) = n. [Hint: Use an indirect proof.]
Bonus: Generalize this problem and prove your generalization.