Math 240 Problem Set #11 M. Flashman

---

1. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5x + 4$.
   a. Prove f is a one to one function.
   b. Prove f is an onto function.
   c. Prove $f([0,1]) = [4,9]$.
   d. Prove $f^{-1}([0, 14]) = [-4/5,2]$.

2. Generalize(unify) the work in parts a and b of Problem #1 for any linear function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = mx + b$ where $m \neq 0$. Discuss briefly the significance of the condition on m.

---

3. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 5x^2 + 4$.
   a. Prove f is **not** a one to one function.
   b. Prove f is **not** an onto function.
   c. Prove $f([0,1]) = [4,9]$.
   d. Prove $f^{-1}([0, 24]) = [-2,2]$.

4. Generalize (unify) the work in parts a and b of Problem #3 for any quadratic function $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = Ax^2 + Bx + C$ where $A \neq 0$.

---

5. Consider $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^3 + mx + 5$.
   a. Use calculus to prove that if $m = 1$ then f is a one to one function.
   b. Use calculus to prove that if $m = -1$ then f is **not** a one to one function.
   c. BONUS: Prove that f is a one to one function **if and only if** $\mathbf{m \geq 0}$.
