Math 240 Fall. 2006

Equivalence Relation Example

Let S = { (a,b) : a and b $\in$**Z,** b $\neq$0}.
Define the relation R on S by (a,b)R(c,d) is true if and only ad = bc
Proposition: R is an equivalence relation on S.

Proof:
i. For any (a,b) $\in$S, ab = ba so by the definition, (a,b)R(a,b), so R is reflexive.
ii. Suppose (a,b) and (c,d) are elements of S and (a,b) R (c,d) is true. Then by definition, ad= bc. Thus cb = da and by definition, (c,d)R (a,b) is true. Thus R is symmetric.

iii. Suppose (a,b), (c,d), and (e,f) are elements of S and (a,b) R (c,d) and (c,d)R(e,f) are true. Then by definition, ad= bc (*) and cf = de (**).
Case 1: If a = 0 then since b$\neq$0, c = 0. And since d$\neq$0, e = 0. Thus af = 0 = be and (by definition) (a,b)R(e,f).
Case 2. If a $\neq$0, then c$\neq$0 and d$\neq$0. Now adf = bcf (from *) and bcf =bde. Now using the transitivity of "=", we have adf = bde and since d$\neq$0, af = be and (by definition) (a,b)R(e,f).
Thus (in either case) we have shown that (a,b)R(e,f) and R is transitive.

Since we have shown r is reflexive, symmetric and transitive, R is an equivalence relation on S.
EOP.