Visualizing Partial Derivatives without Graphs.

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Abstract

- In this presentation the author will
  - explain and use **free graphing technology** (**Winplot**) to illustrate
  - how to **visualize the partial derivative without graphs**.
- The treatment is suitable for any introductory treatment of the concept.
- **Based on mapping (transformation) figures** this approach
  - allows students to understand the concepts in an n-dimensional context
  - without any change in presentation from that given for the ordinary derivative.

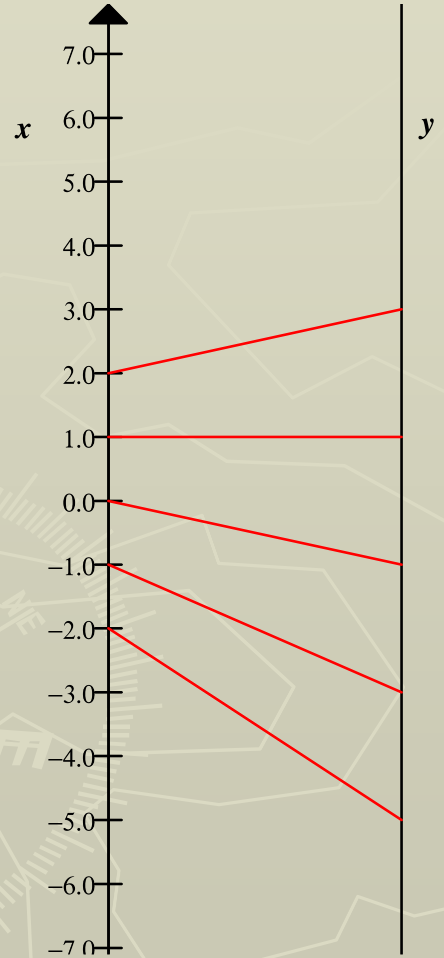
Foundations
Mapping (Transformation) Figures

- Visualize functions $f: R \rightarrow R$

$$y = f(x)$$

- Winplot examples:
  - linear
  - nonlinear

$y = 2x - 1$



7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
−1.0
−2.0
−3.0
−4.0
−5.0
−6.0
x
y

$y = x^2$

7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
−1.0
−2.0
−3.0
−4.0
−5.0
−6.0
x
y

Dynamic interpretation

- Dynamic interpretation of the derivative visualized using
  - $\partial y/\partial x$
  - rates
- Illustrate using Winplot

Visualizing Multi-Variable Functions

- Visualize functions f: $R^n \rightarrow R$

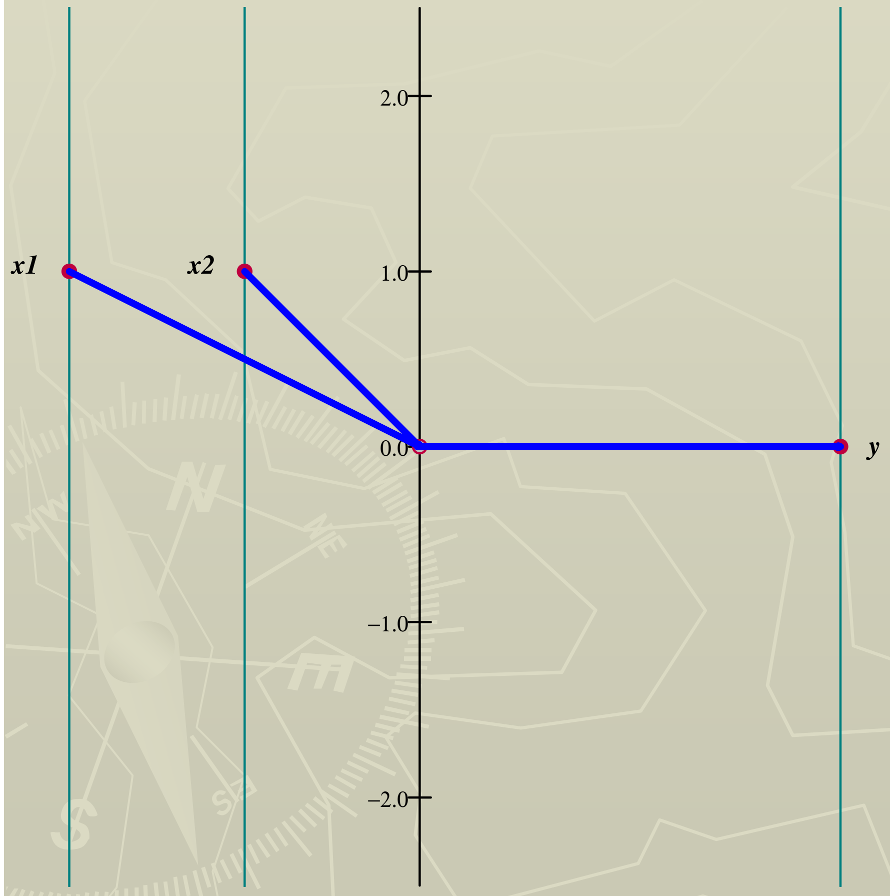
$$y = f(x_1, x_2, \ldots, x_n)$$

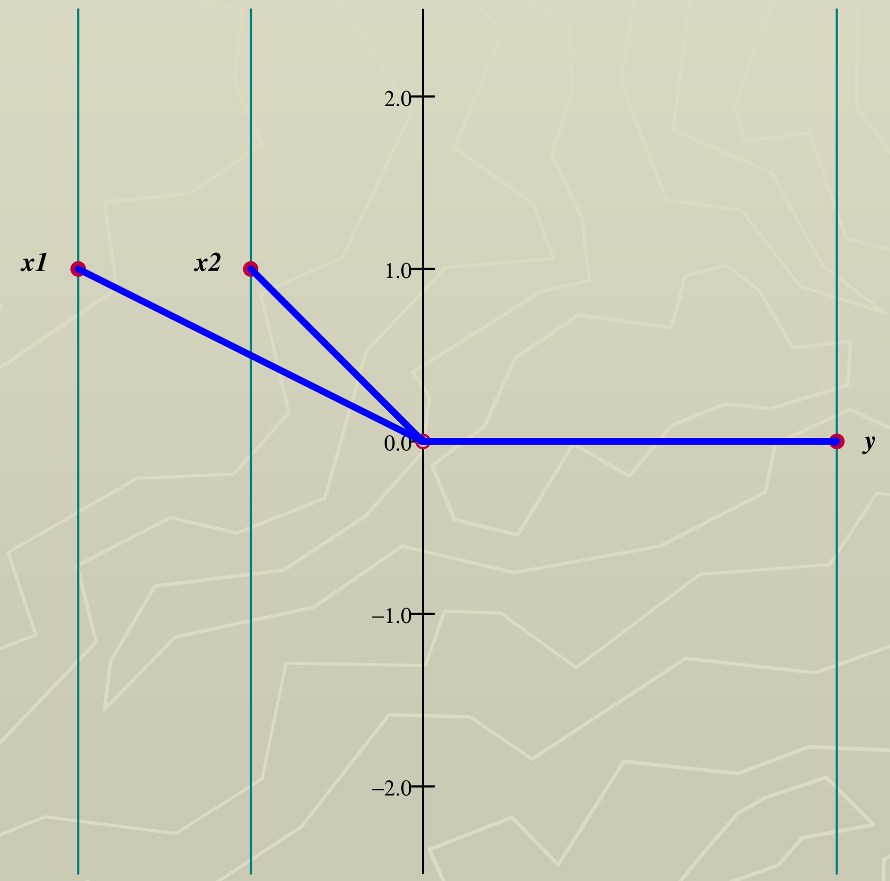
- Visualize f: $R^2 \rightarrow R$

$$y = f(x_1, x_2)$$

$y = 2(x_1-1) - 3(x_2-1)$

$y = x_1^2 - x_2^3$





Visualizing The Partial Derivative for $y = f(x_1,x_2)$

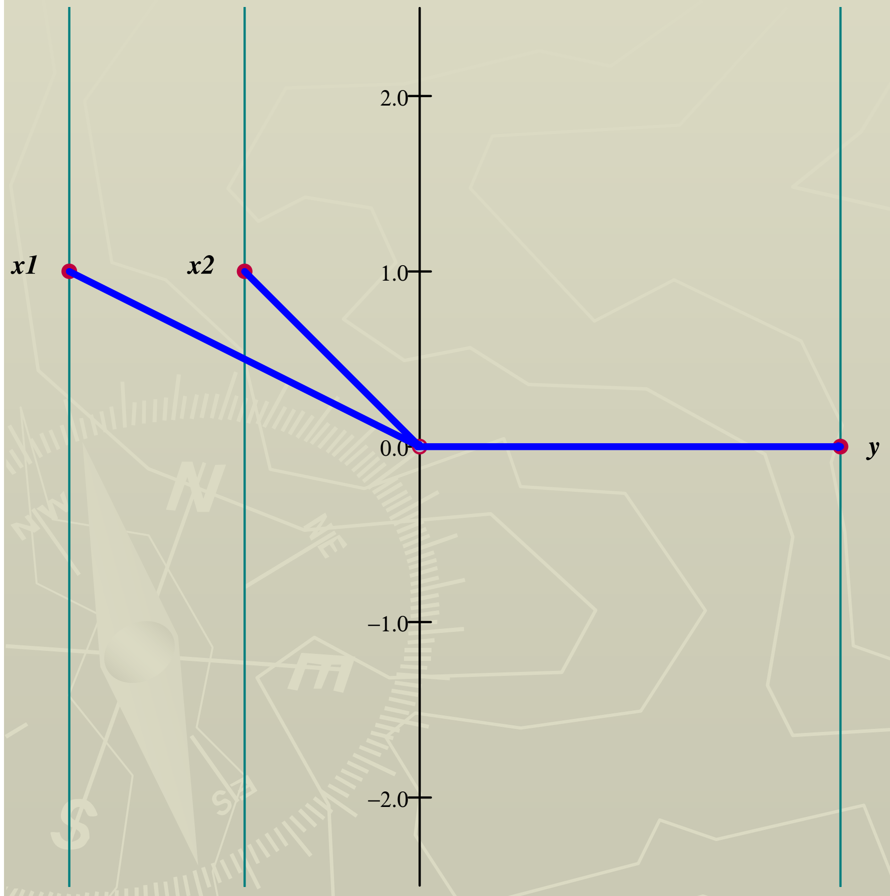
- Dynamic interpretation of the partial derivatives for $f: R^2 \rightarrow R$

$$y = f(x_1,x_2)$$

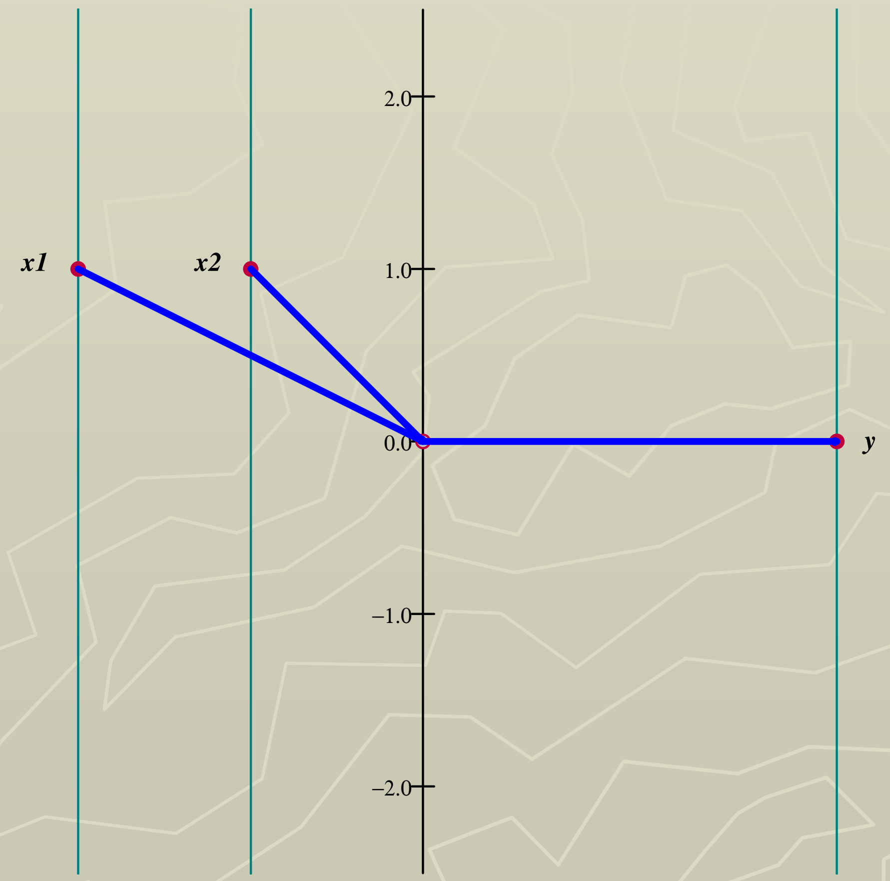
visualized using $\partial y/\partial x_1$ and $\partial y/\partial x_2$.

- Static picture next slide.
- Winplot dynamic visualization.

$y = 2(x_1-1) - 3(x_2-1)$



$y = x_1^2 - x_2^3$



Visualizing The Partial Derivative for $y = f(x_1,x_2,\ldots,x_n)$

- Dynamic interpretation of the partial derivative for $f: R^n \to R$

  $$y = f(x_1,x_2,\ldots,x_n)$$

  visualized using $\partial y/\partial x_1, \ldots, \partial y/\partial x_n$.
- Static picture here.
- Winplot dynamic visualization.

Conclusion

- Can use this visualization for other aspects of functions
- $f: R^n \rightarrow R^k$

$$f(x_1,x_2,\ldots,x_n)=(y_1,y_2,\ldots,y_k)$$

where $y_k = f_k(x_1,x_2,\ldots,x_n)$

Time!

- Questions?
- Responses?
- Further Communication by e-mail:

  flashman@humboldt.edu

- These notes will be available at

http://www.humboldt.edu/~mef2

Thanks-

The end!