Exploring Linear Functions in The Common Core with Mapping Diagrams

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in The Common Core
with Mapping Diagrams
Links:

www.edmodo.com/flashman/referral

Martin Flashman's Sensible Mathematics
Group Code: muj2ak

https://edmodo.com/public/martin-flashman-039-s-sensible-mathematics/group_id/14257655

http://users.humboldt.edu/flashman/Presentations/NMP/NMP.MD.LINKS.html

The Two Most Important Mathematical Concepts!

Number	**Function**
K-8	6-8
Counting	Equations
Fractions	Proportionality
Integers	Rates
Rationals	Linear Equations/ Functions
Algebraic	Quadratic Equations/Functions
Reals	

Common Core Connections (Grade 6)

- Understand **ratio** concepts and use ratio reasoning to solve problems.
  - **Use ratio and rate reasoning to solve real-world and mathematical problems**, e.g., by reasoning about tables of equivalent ratios, tape diagrams, **double number line diagrams**, or equations.
- Find and **position integers and other rational numbers on a horizontal or vertical number line diagram**; find and position pairs of integers and other rational numbers on a coordinate plane.

Common Core Connections (Grade 7)

- **Analyze proportional relationships** and use them to solve real-world and mathematical problems.
  - **Identify the constant of proportionality (unit rate) in** tables, graphs, equations, **diagrams**, and verbal descriptions of proportional relationships.

- Apply and extend previous **understandings of addition and subtraction** to add and subtract rational numbers; represent addition and subtraction **on a horizontal or vertical number line diagram.**

Common Core Connections (Grade 8)

- Define, evaluate, and compare functions.
  - **Understand that a function is a rule that assigns to each input exactly one output.** The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
  - **Compare properties of two functions each represented in a different way** (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function …. determine which function has the greater rate of change.
  - **Interpret the equation y = mx + b as defining a linear function**, whose graph is a straight line; give examples of functions that are not linear.

Common Core Connections (Grade 8)

- Use functions to model relationships between quantities..
  - Construct a function to model a linear relationship between two quantities. **Determine the rate of change and initial value of the function** from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. **Interpret the rate of change and initial value of a linear function** in terms of the situation it models, and in terms of its graph or a table of values.
  - **Describe qualitatively the functional relationship between two quantities** by analyzing a graph (e.g., **where the function is increasing or decreasing, linear or nonlinear**).

Common Core Connections (Functions)

Functions Overview

Interpreting Functions

Understand the concept of a function and use function notation

Interpret functions that arise in applications in terms of the context

Analyze functions using different representations

Building Functions

Build a function that models a relationship between two quantities

Build new functions from existing functions

Linear, Quadratic, and Exponential Models

Construct and compare linear and exponential models and solve problems

Interpret expressions for functions in terms of the situation they model.

Common Core Connections

(Functions)

Mathematical Practices

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- **Construct viable arguments** and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- **Look for and make use of structure.**
- **Look for and express regularity in repeated reasoning.**

Background Questions

- Are you familiar with Mapping Diagrams to visualize functions?
- Have you used Mapping Diagrams to teach functions?
- Have you used Mapping Diagrams to teach content besides function definitions?

Main Resource

- Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s. A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET FOR publication)

- http://users.humboldt.edu/flashman/MD/section-1.1VF.html

Visualizing Linear Functions

- Linear functions are a basic element of the Common Core - even without considering their graphs.
- There is a sensible way to visualize them using "mapping diagrams."
- Examples of <u>important Common Core function features (like rate and intercepts)</u> can be illustrated sensibly with mapping diagrams.
- Activities for students using mapping diagrams engage understanding <u>for both function and linearity concepts</u>.
- Creating Mapping diagrams can use simple tools (straight edges) as well as technology.

Linear Mapping diagrams

We begin our more detailed introduction to linear functions using mapping diagrams :

“ $y = f(x) = mx + b$ ”

Distribute Worksheet *now.*

Do Problem 1

Prob 1: Linear Functions -Tables

x	5 x - 7
3	
2	
1	
0	
-1	
-2	
-3	

Complete the table.

x = 3,2,1,0,-1,-2,-3

$f(x) = 5x - 7$

Linear Functions: Tables

x	5 x – 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

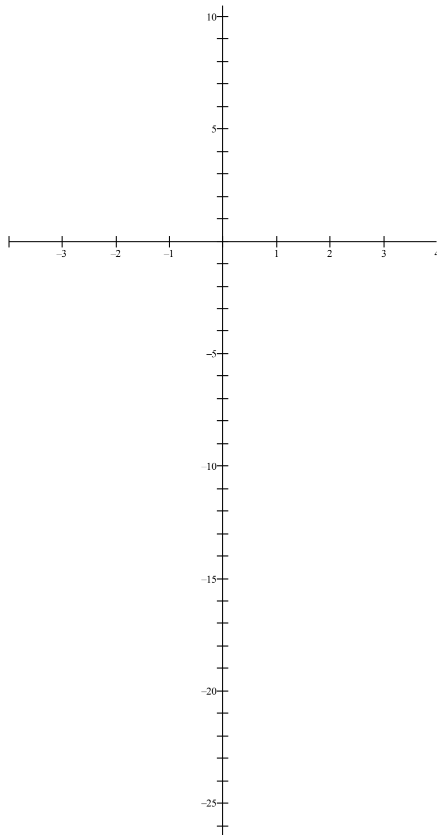
Complete the table.

$x = 3,2,1,0,-1,-2,-3$

$f(x) = 5x - 7$

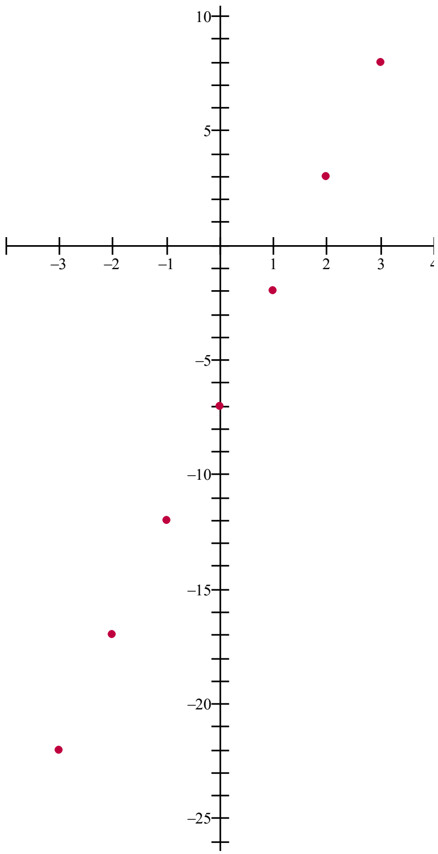
Linear Functions: On Graph

Plot Points (x , 5x - 7):



X	5 x - 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

Linear Functions: On Graph

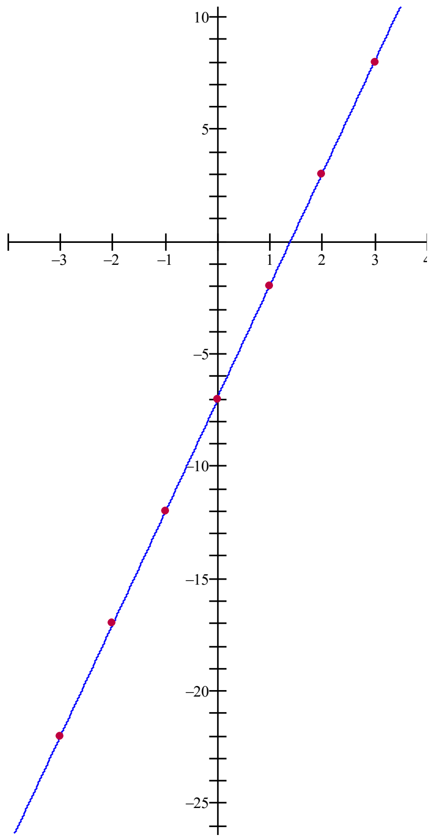


Connect Points

(x , 5x - 7):

X	5 x – 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

Linear Functions: On Graph



Connect the Points

X	5 x - 7
3	8
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1	-2
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Linear Functions: Mapping diagrams Visualizing the table.

- Connect point x to point 5x – 7 on axes

X	5 x – 7
3	8
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Linear Functions: Mapping diagrams Visualizing the table.

- Connect point x to point 5x – 7 on axes

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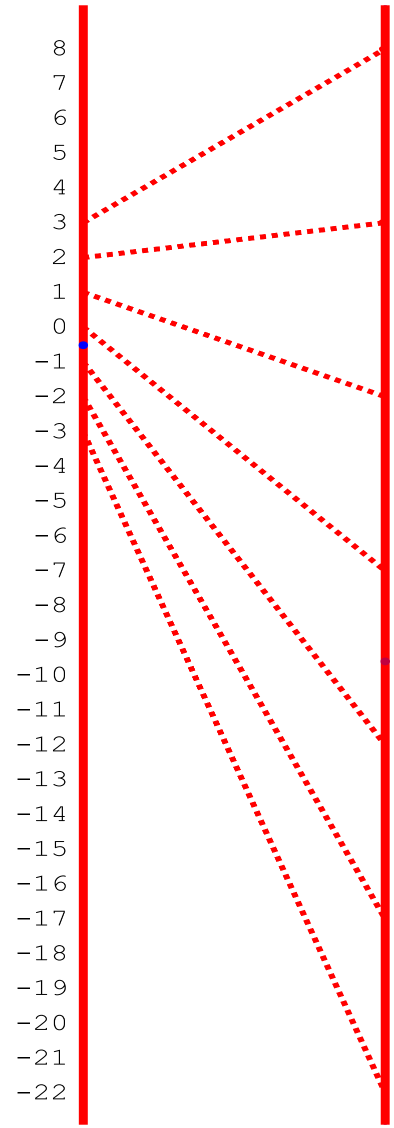

Technology Examples

- Excel example
- Geogebra example

Linear Functions
Core Examples

$m(x) = mx; \quad m=5$

$s(x) = x + b; \quad b=-7$

$$\begin{aligned} f(x) &= s(m(x)) \\ &= mx + b; \\ &= 5x - 7 \end{aligned}$$

Table and Mapping Diagram: $m(x) = 5x$

x	$m(x) = 5x$
2	10
1	5
0	0
-1	-5
-2	-10

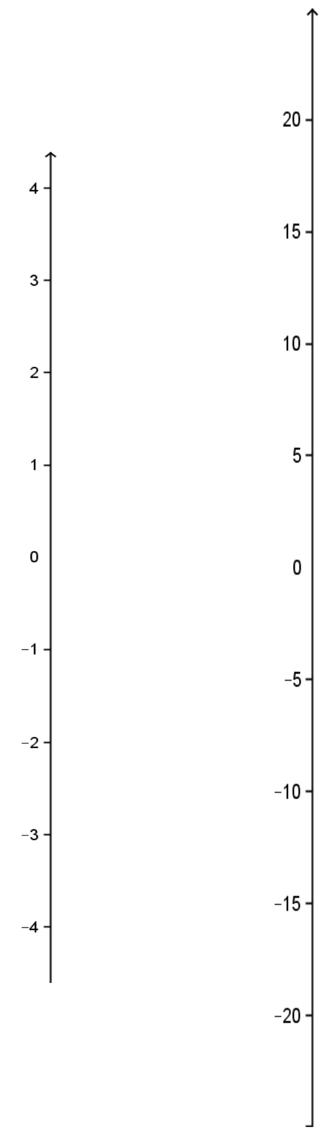

Table and Mapping Diagram: $m(x) = 5x$

x	m(x) =5x
2	10
1	5
0	0
-1	-5
-2	-10

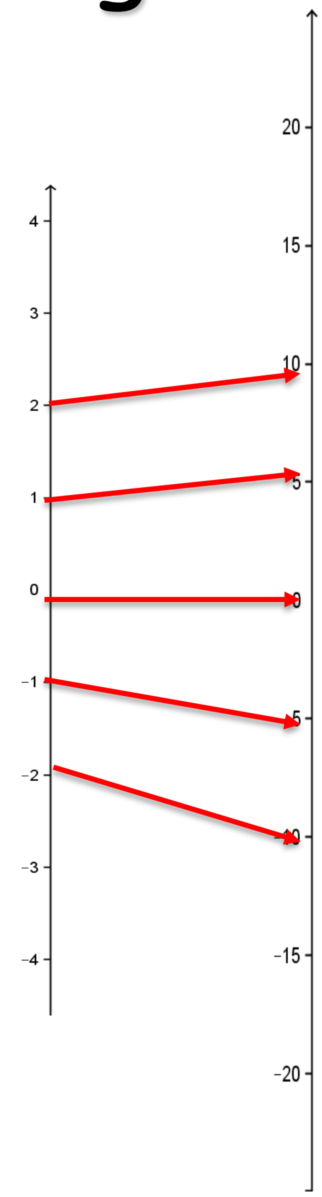

Table and Mapping Diagram:
$s(x) = x - 7$

x	$s(x) = x-7$
10	3
5	-2
0	-7
-5	-12
-10	-17

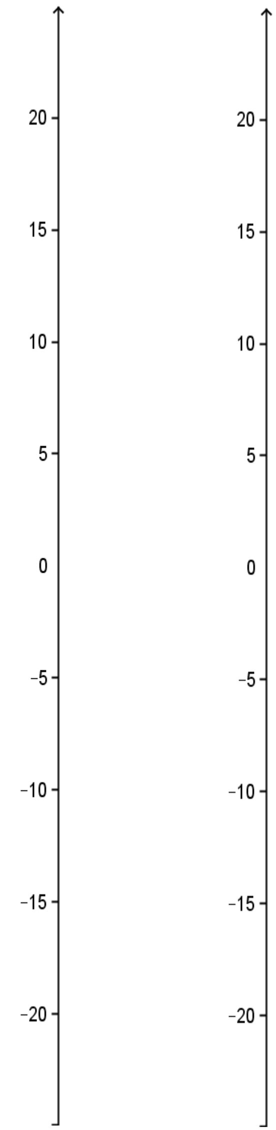

Table and Mapping Diagram:

$s(x) = x - 7$

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10	3
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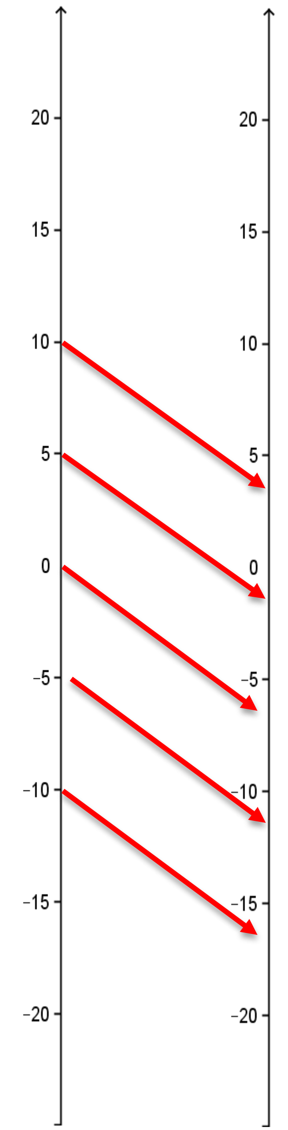
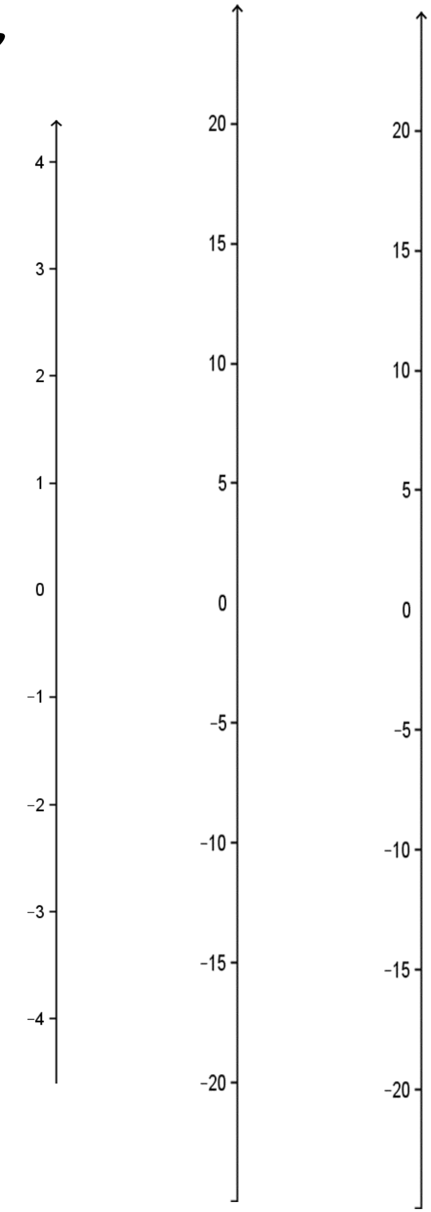


Table and mapping diagram
$f(x) = s(m(x)) = 5x - 7$

x	m(x)	f(x)=s(m(x))
2	10	3
1	5	-2
0	0	-7
-1	-5	-12
-2	-10	-17



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2	10	3
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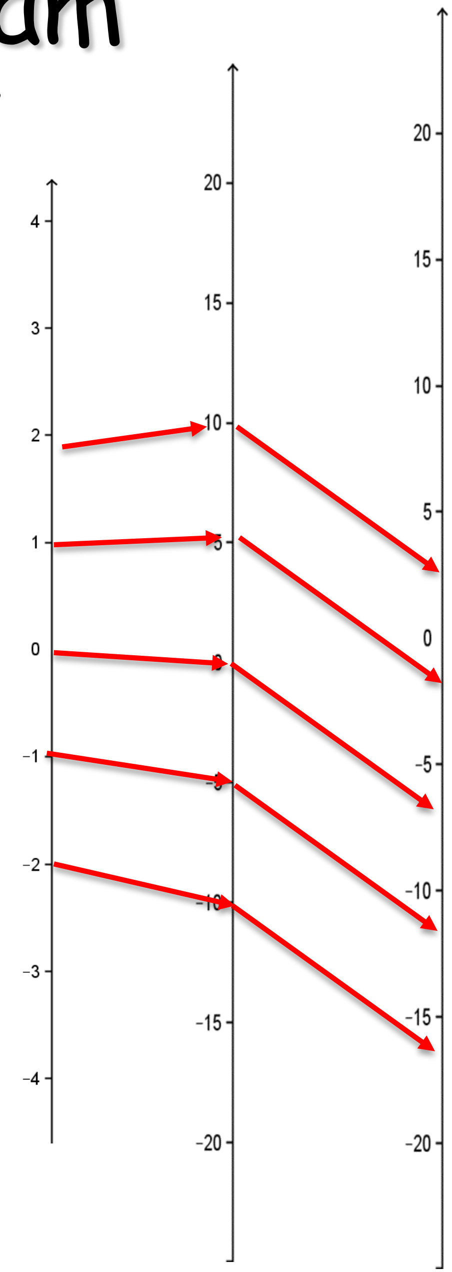

Table and mapping diagram

$$f(x) = s(m(x)) = 5x - 7$$

x	m(x)	f(x)=s(m(x))
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1	5	-2
0	0	-7
-1	-5	-12
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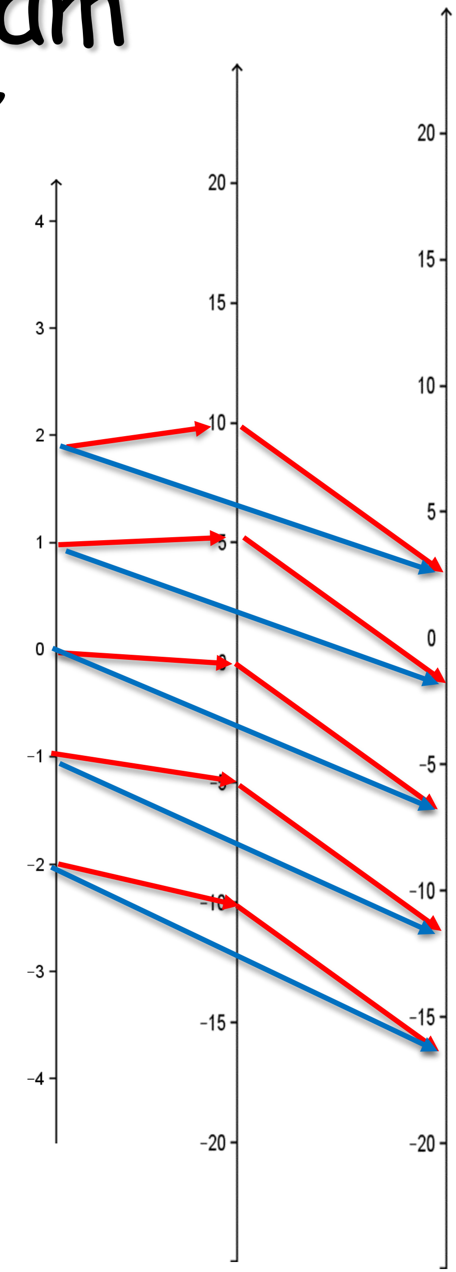


More on Linear Mapping diagrams

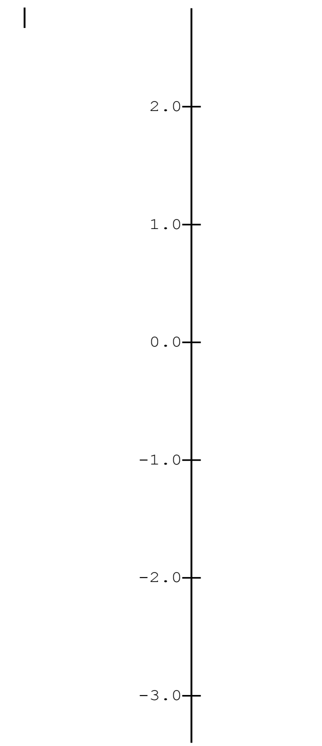
We continue by a consideration of the **composition of linear functions**.

Do Problem 5

Problem 5: Compositions are keys!

An example of composition with mapping diagrams of simpler (linear) functions.

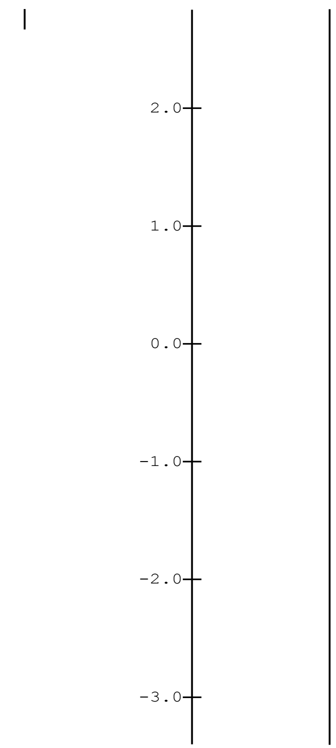
- $g(x) = 2x$; $h(x)=x+1$
- $f(x) = h(g(x)) = h(u)$
  where $u = g(x) = 2x$
- $f(x) = (2x) + 1 = 2x + 1$
  $f(0) = 1$ $m = 2$



Compositions are keys!

All Linear Functions can be understood and visualized as compositions with mapping diagrams of simpler linear functions.

- $f(x) = 2\,x + 1 = (2x) + 1$:
  - $g(x) = 2x$; $h(u)=u+1$
  - $f\,(0) = 1$ $m = 2$



Simple Examples are important!

$f(x) = mx + b$ with a mapping diagram --
Five examples:
Back to Worksheet Problem #2

- Example 1: $m = -2$; $b = 1$: $f(x) = -2x + 1$
- Example 2: $m = 2$; $b = 1$: $f(x) = 2x + 1$
- Example 3: $m = \frac{1}{2}$; $b = 1$: $f(x) = \frac{1}{2}x + 1$
- Example 4: $m = 0$; $b = 1$: $f(x) = 0x + 1$
- Example 5: $m = 1$; $b = 1$: $f(x) = x + 1$

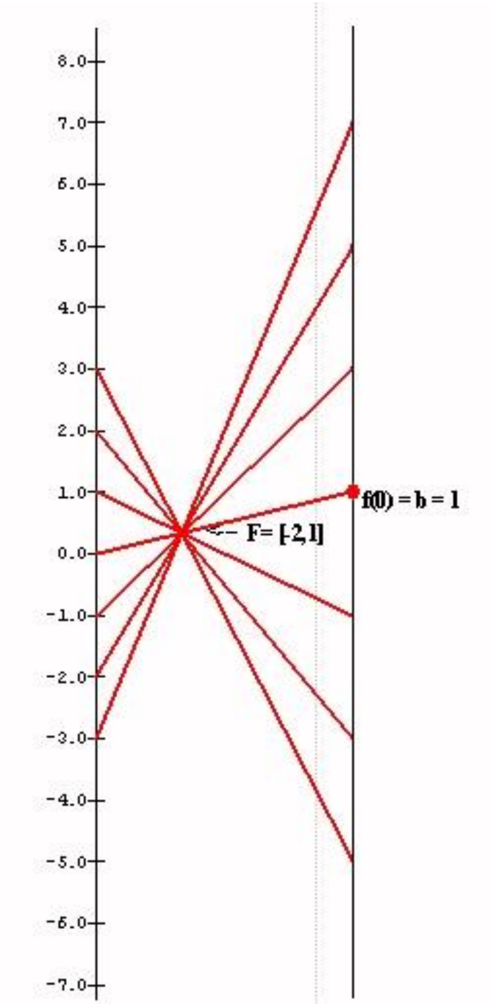
Visualizing $f(x) = mx + b$ with a mapping diagram -- Five examples:

Example 1: m = -2; b = 1

f (x) = -2x + 1

- Each arrow passes through a single point, which is labeled **F = [- 2,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $-2x + 1$,

    which corresponds to the linear function's value for the point/number, *x*.



Visualizing *f* (*x*) = ***mx*** + ***b*** with a mapping diagram -- Five examples:

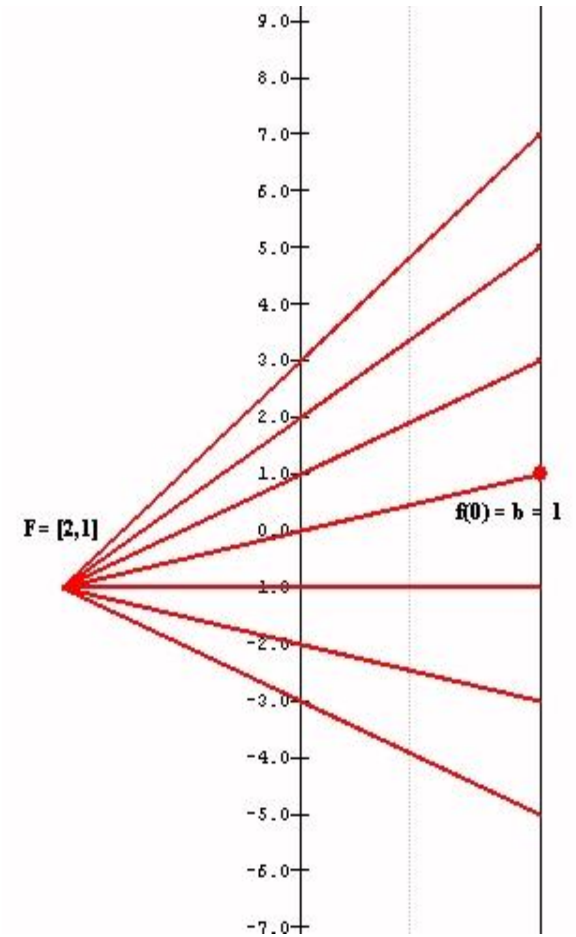
Example 2: *m* = 2; *b* = 1
f(x) = 2x + 1

Each arrow passes through a single point, which is labeled

F = [2,1].

- The point **F** completely determines the function *f*.
  - **given a point** / number, *x*, on the source line,
  - there is a **unique arrow passing through F**
  - **meeting** the target line at **a unique point** / number, 2*x* + 1,

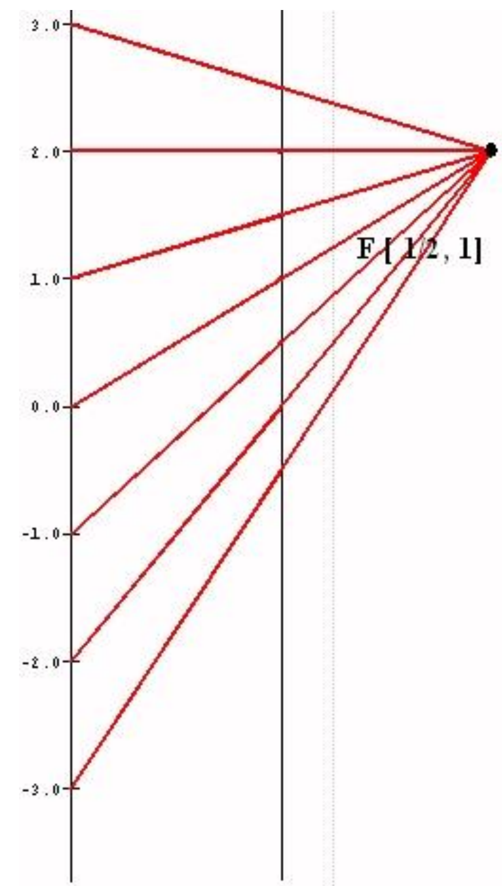
  which corresponds to the linear function's value for the point/number, *x*.



Visualizing $f(x) = mx + b$ with a mapping diagram -- Five examples:

- **Example 3: *m* = 1/2; *b* = 1**
  $f(x) = \frac{1}{2} x + 1$
- Each arrow passes through a single point, which is labeled **F = [1/2,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $\frac{1}{2}x + 1$,

    which corresponds to the linear function's value for the point/number, *x*.

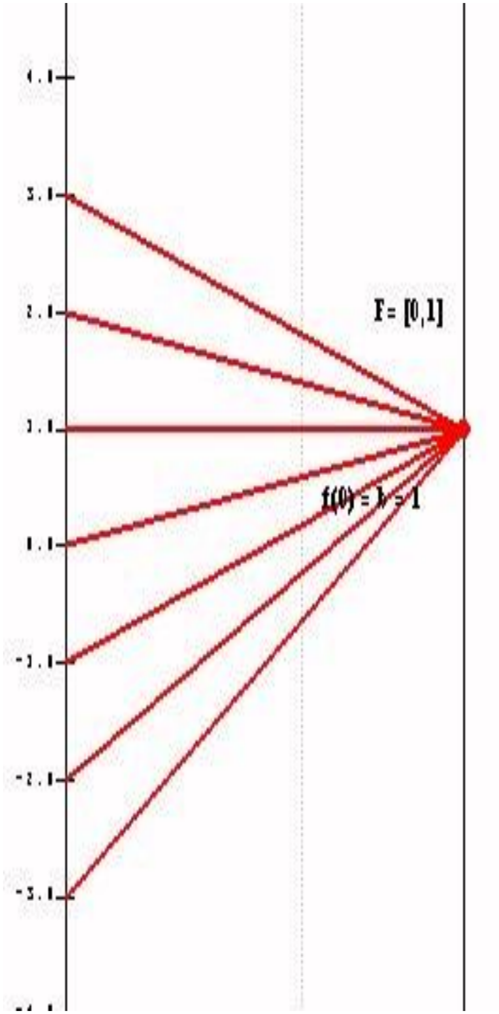


Visualizing $f(x) = mx + b$ with a mapping diagram -- Five examples:

- **Example 4: $m = 0$; $b = 1$**
  $f(x) = 0\,x + 1$

- Each arrow passes through a single point, which is labeled **F = [0,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $f(x)=1$,

    which corresponds to the linear function's value for the point/number, *x*.



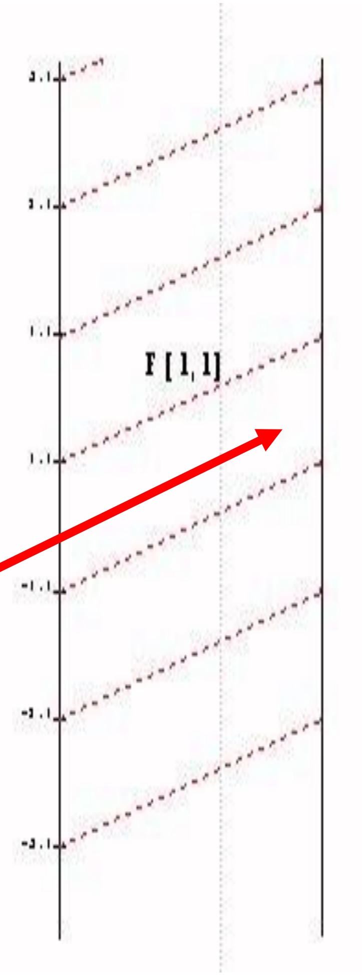
Visualizing $f(x) = mx + b$ with a mapping diagram -- Five examples

Example 5: $m = 1$; $b = 1$

$f(x) = x + 1$

- Unlike the previous examples, in this case it is not a single point that determines the mapping diagram, but the single arrow from 0 to 1, which we designate as F[1,1]
- It can also be shown that this single arrow completely determines the function. Thus, given a point / number, x, on the source line, there is a unique arrow passing through ***x parallel to*** F[1,1] meeting the target line a unique point / number, $x + 1$, which corresponds to the linear function's value for the point/number, x.
  - The single arrow completely determines the function f.
    - **given a point** / number, x, on the source line,
    - there is a **unique arrow** through ***x parallel to*** F[1,1]
    - **meeting** the target line at **a unique point** / number, $x + 1$,

which corresponds to the linear function's value for the point/number, x.



Simple Examples are important!

- $f(x) = x + C$ Added value: C
- $f(x) = mx$ Scalar Multiple: m

  Interpretations of m:
  - slope
  - rate
  - Magnification factor
  - $m > 0$: Increasing function
  - $m < 0$: Decreasing function
  - $m = 0$: Constant function

Function-Equation Questions
Simple Linear Equations

$$5x - 7 = 8$$

Function-Equation Questions

Simple Linear Equations and Linear Functions

Linear Equations

$5x - 7 = 8$

$\underline{+7 = +7}$

$5x = 15$

$1/5(5x) = 1/5(15)$

$x = 5$

Linear Functions

$f(x) = 5x - 7$

$\underline{m(x) = 5x;\ s(x) = -7}$

$f(x) = s(m(x))$

Function-Equation Questions

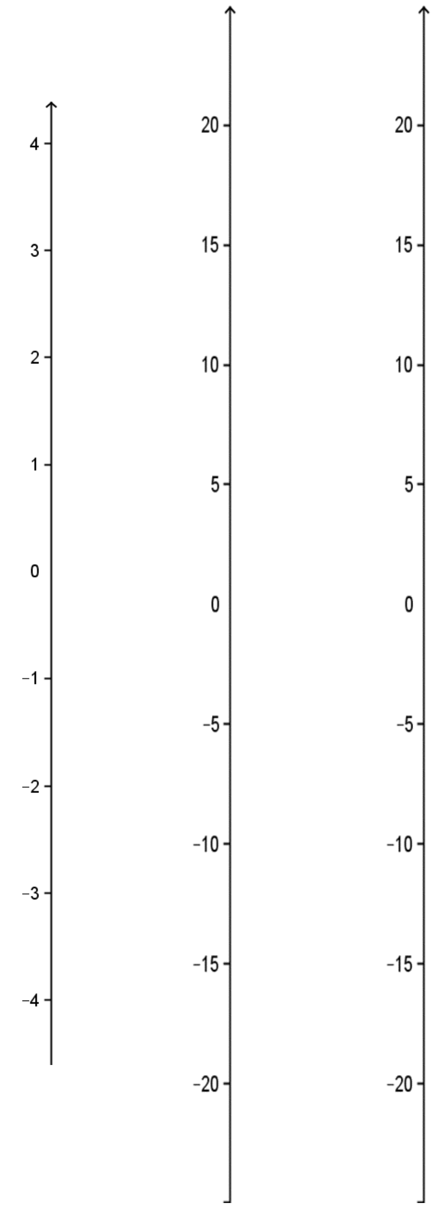
Simple Linear Functions

$f(x) = 5x - 7$

$\underline{m(x) = 5x}$ $\underline{s(x) = x - 7}$

$f(x) = s(m(x))$

x	m(x)	f(x)=s(m(x))
2	10	3
1	5	-2
0	0	-7
-1	-5	-12
-2	-10	-17



Function-Equation Questions

with linear focus points (Problem 3)

- Solve a linear equation:

  $2x+1 = 5$

  – Use focus to find x.

Function-Equation Questions

with linear focus points (Problem 4)

Suppose f is a linear function with f (1) = 3 and f (3) = -1.

- Without algebra
  - Use focus to find f (0).
  - Use focus to find x where f (x) = 0.

Thanks
The End!

☺

Questions?

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http://users.humboldt.edu/flashman

References

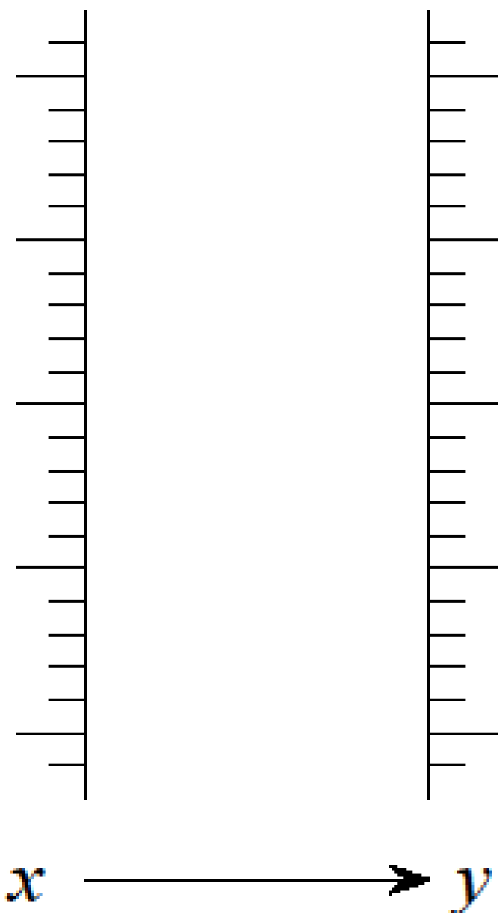
Mapping Diagrams and Functions

- **Function Diagrams. by Henri Picciotto Excellent Resources!**
  - **Henri Picciotto's Math Education Page**
  -
- Flashman, Yanosko, Kim https://www.math.duke.edu//education/prep02/teams/prep-12/

Function Diagrams by Henri Picciotto

Function Diagrams

Henri Picciotto, www.picciotto.org/math-ed



x	y

More Think about These Problems

M.1 How would you use the Linear Focus to **find the mapping diagram for the function inverse for a linear function when m≠0?**

M.2 How does the **choice of axis scales** affect the **position of the linear function focus point** and its use in solving equations?

M.3 Describe the visual features of the mapping diagram for the quadratic function $f(x) = x^2$.
How does this generalize for *even* functions where
$$f(-x) = f(x)?$$

M.4 Describe the visual features of the mapping diagram for the cubic function $f(x) = x^3$.
How does this generalize for *odd* functions where
$$f(-x) = -f(x)?$$

Thanks

The End!   REALLY!

☺

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