Visualizing Functions with Mapping Diagrams in GeoGebra

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ICTCM
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Links:

http://users.humboldt.edu/flashman/Presentations/ICTCM15/MD.LINKS.html

Outline

I. Background and Links

II. Example: Getting Started with GG: Duality.

III. Example: A Diagram for A Linear Function

IV. Playing with The Linear Example.

   A. Bells and Whistles.

   B. Graphs

V. Learning by Deconstruction.

   A. Example: Henri Picciotto's Sampler.

   B. Example: Composition and Inverses.

   C. Example: The Derivative

   D. Other Topics : Choices!

I. Background and Links

Background Questions

- Are you familiar with Mapping Diagrams?
- Are you familiar with GeoGebra?
- New? Novice/Beginner? Pair up with
- Intermediate/Expert.

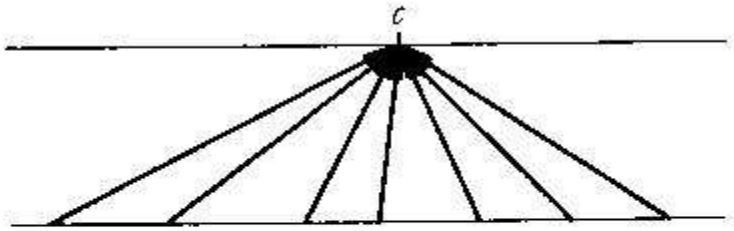
Mapping Diagrams (MD)

A.k.a.

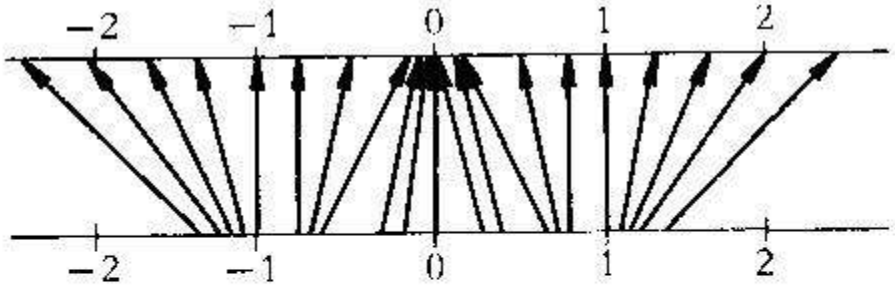
Function Diagrams

Dynagraphs

Two Figures from Ch. 5 *Calculus* by M. Spivak



(a) $f(x) = c$



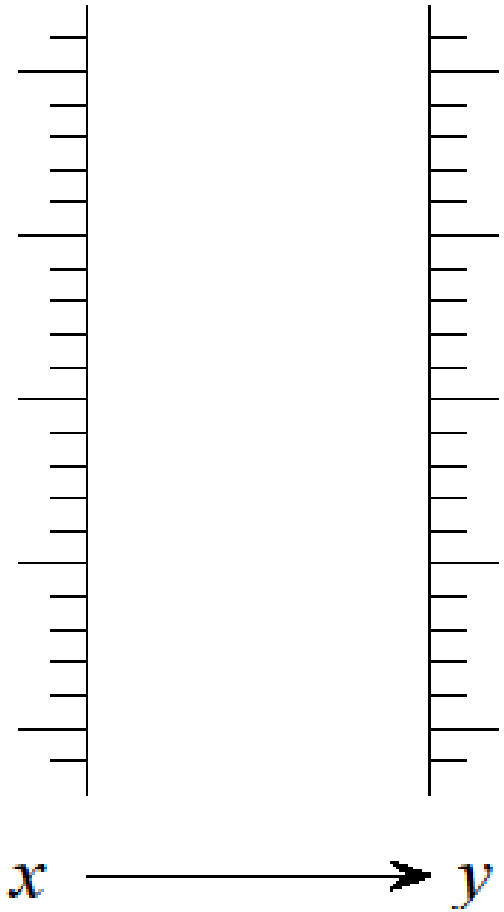
(b) $f(x) = x^3$

FIGURE 2

Function Diagrams by Henri Picciotto

Function Diagrams

Henri Picciotto, www.picciotto.org/math-ed



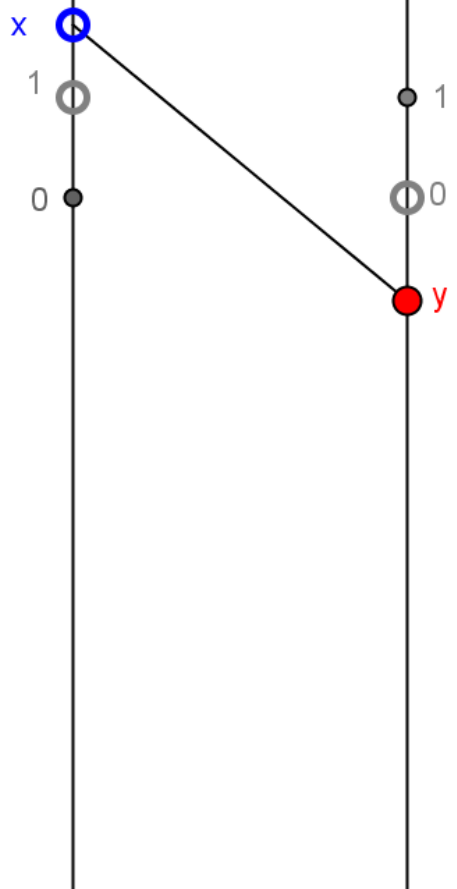
x	y

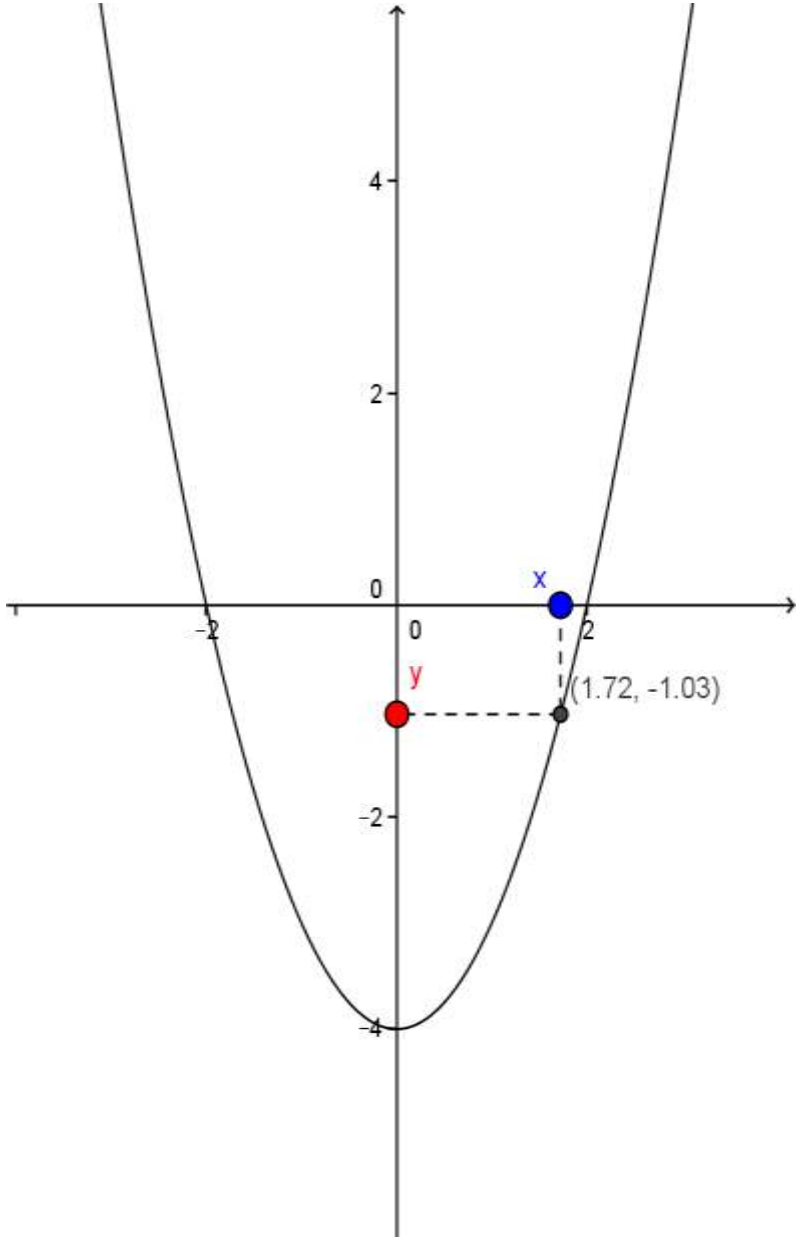
From: Function Diagram Applets

by Henri Picciotto

Henri Picciotto's Math Education Page

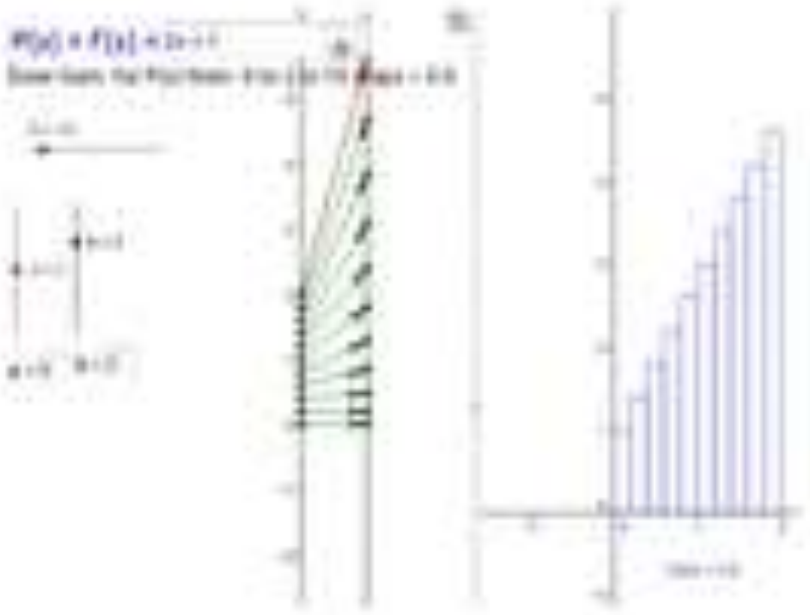
http://www.mathedpage.org/

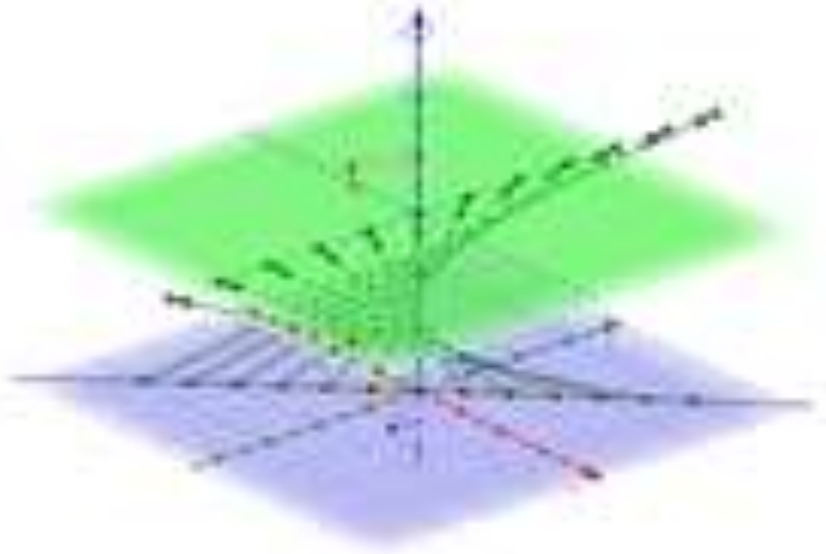




More Examples of MD's

GeoGebraBook: Mapping Diagrams Take on Calculus and Complex Variables





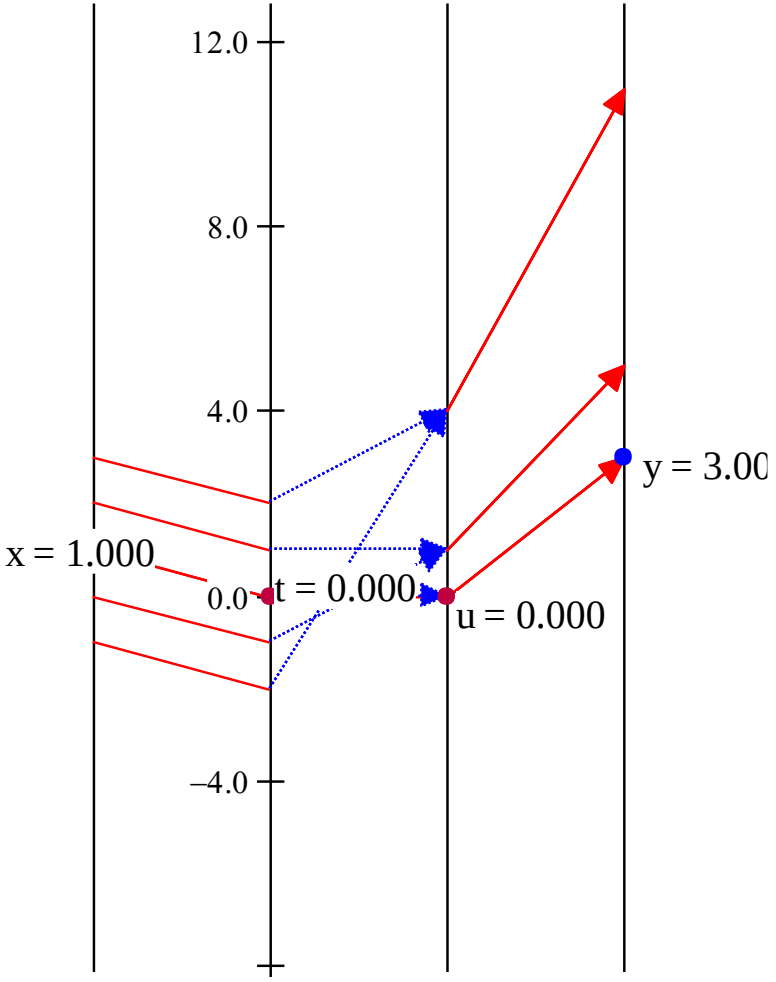
How Mapping Diagrams Work: An Example of Composition

A Goal: Create similar MD's.

$g(x) = 2(x-1)^2 + 3$

Steps for *g*:

1. **Linear: Subtract 1.**
2. **Square result.**
3. **Linear: Multiply by 2 then add 3.**



II. First Steps with GG: Duality in Two Frames

A Main Resource for MD's

- Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s. A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET published)

http://users.humboldt.edu/flashman/MD/section-1.1VF.html

Basic Tools for GeoGebra (GG)

Resources from GeoGebra

Main Page for GeoGebra: http://geogebra.org/

More links at end of presentation.

Beginning GG: Basic Tools

* available at opening of GG

- Views: Toolbar [Tools/Commands/Functions and Operators]
  - Algebra View *
  - Graphics View * [Actually two graphic frames]
  - Spreadsheet View
  - CAS View
  - 3D_Graphics View
  - Probability Calculator
- Other Components:
  - Menubar*
  - Input Bar *

GeoGebra Basics for Mapping Diagrams: Two Frames

- **Two Frames: View… Graphics 2**
- **Preferences: View… Layout… Graphics 2… Preferences- (2) … xAxis… uncheck boxes for**

❑ **Show xAxis and**

❑ **Show Numbers.**

GeoGebra Basics for Mapping Diagrams: Create Parallel Axes

- **Create Parallel Axis in Graphics 2:**
  <u>Click on Graphics 2…</u>
  Input "l: x = l_a"
  Response : "Create slider(s) for: l_a"
  In Algebra appears: Line l:x=1
  Number l_a = 1
  In Graphics 2 appears:
  Horizontal slider for l_a = 1.
  Move slider to 2.
  In Algebra:
  Click on number "l_a = 2"… slider is hidden.

GeoGebra Basics for Mapping Diagrams: **Points and Vectors**

- **In Graphics: Use point tool to create free point**
  Algebra: *A* = (…,…) in Graphics point labelled "*A*".

- **Click on Graphics 2…** **Input:**
  "X=Point[yAxis]" **Point on yAxis**
  "Y=Point[l]" **Point on line l**
  "XY=Vector[X,Y]" **Vector from X to Y**
  "X_A=(0, x(A))" **y(A): "x"-coordinate of A**
  "Y_A=(l_a,y(A))" **y(A): "y"-coordinate of A**
- **"V_A=Vector[X_A,Y_A]"**
- **Click on Graphics … Input:**
  "P_{XY} =(y(X),y(Y))"

Experiment: Move A, X, and Y. Observations?

Why is this an example of "duality"?

III. Example: Building A Diagram for A Linear Function

Linear Mapping Diagrams
An Introduction

We begin our more detailed introduction to building mapping diagrams with GeoGebra:

We will create a mapping diagram for linear functions of the form

" $y = f(x) = mx + b$ "

GeoGebra Basics for Mapping Diagrams: **Numbers and Functions**

- **Numbers: "b=1" "m=-2":** May be entered as input.
  This will create a hidden "slider" shown on the Algebra View with a name and current value. Also can be created using Slider tool or when "needed" for other inputs.
  [Unless specified – GGB names (lower case letters) are by default.]
  Click on bullet for Number: Shows slider in Graphics View.
  Many other features can be controlled by **object properties.**
- **Functions: "f(x) = m*x+b"** Entered as input. This will create a function in the Algebra View shown on the Graphics View with its name.
  Clicking on bullet for Function hides graph of function in Graphics View.
  Clicking on function or its graph in the Graphics View or in Algebra View with Graphics Features open to quickly change some properties [Name and value].
  Many other features can be controlled by object properties.

GeoGebra Basics for Mapping Diagrams: **Points**.

- **Points: On Graphics 2 "F_x=(l_a, f(y(X))"**

  This will create a point in the Algebra View shown on the Graphics View with its name. [Convention: **Capitals for points.]**

  Create Captions: x = %y" - %y gives y coordinate of current point for X. "f(x) = %y" - %y gives y coordinate of current point for F_x.

GeoGebra Basics for Mapping Diagrams: Vectors, Text, and Graphics.

- **Vectors: "v_f=Vector[X, F_x]"**

  May be entered as input or by using vector tool (two points and click) .

This will create a vector in the Algebra View shown on the Graphics View with its name.

Click on vector in the Graphics View or in Algebra View with Graphics Features open to quickly change some properties. [e.g. Hide label]
Many other features can be controlled by object properties.

- **Text:"f(x) =[f]\\ f([y(X)])=[y(F_x)] "**

  May be entered using Text tool on Graphics.

This will create a text box shown on the Graphics View – allows LaTeX formatting and dynamic data [Object] evaluation.

III. Playing with The Linear Example

Ready to play? A

- Play with the basic example:
  - Move x.
  - Change m with slider.
    - Increasing/Decreasing?
    - Initial value vs. zero.
  - Animation on for "x" :
    - In Graphics or Algebra
    - Adds animation button to Graphics View.
  - Undo and Redo.

Simple Examples are important!

- $f(x) = x + C$ Added value: C
- $f(x) = mx$ Scalar Multiple: m
  Interpretations of m on MD:
  - slope
  - rate
  - Magnification factor
  - $m > 0$: Increasing function
  - $m < 0$: Decreasing function
  - $m = 0$: Constant function

Ready to play? B

- Trace Vector.
  - In Graphics or Algebra
  - Use mouse or arrow key.
  - Increment (and Speed) determined in Algebra properties of X. Change to 1.
    See "m as magnification"

Ready to play? C

- Compare to Graphics 2 View
  - Graph of f on Graphics 2
    Object Properties for f: Advanced – Location.
- **Add points on Graphics:**
  - "P_{x,0}=(y(X),0)"
  - "P_{x,f(x)}=(y(X),y(F_x))"
  - **Move X on Graphics 2.**
- **Add point to graph of f.**
  - "P_f=Point[f]"
- **Add Two points and Vector to MD.**
  - "X_P=(0,x(P_f))"; "F_P=(l_a,y(P_f))";
    "V_P=Vector[X_P,F_P]"
  - **Move P_f on Graphics.**
- **Play with Traces: MD Vector and $P_{x,f(x)}$**

Ready to play? D

- Change colors, fonts, other features.
- Change function:
  - In Algebra
  - With Input Box Tool.

    Caption/ Linked Object: "f(x) =" [f]
  - "m*x^2 + b"- play w/ trace off/on. Notice features.
    - Increasing/Decreasing
    - Minimum/ maximum
    - Different magnification factors.
    - Symmetry
    - Initial value vs. zero.

Ready to play? E

- Lists: Sequences and Zip
  - Sequence[<Expression>,<Variable i>, <Start Value a>, <End Value b>,<Increment>] Zip[<Expression>,<Var1>, <List1>,…]
  - **Create list of numbers** : [New numbers!]
    x_i=Sequence[t, t, x_0, x_n, dt]
  - Create list of points on x-axis using list x_i:
    x_s=**Zip**[(0, t), t, x_i]
  - Create list of function values on y-axis using list x_i:
    y_s=**Zip**[(a, f(t)), t, x_i]
  - Create list of MD vectors from points on x-axis to y-axis:
    v_s=**Zip**[Vector[A, B], A, x_s,B, y_s]
  - Create list of points on graph of f:
    P_s=**Zip**[(A, f(A)), A, x_i]
- **Check Box Tool:** Hide Show
  Creates check box: a Boolean Value attached to objects

Ready to play? F
Function-Equation Question

Hide current work.

Suppose g is a linear function with $g(1) = 3$ and $g(3) = -1$.

Without algebra

- Find "focus" of g using lines.
- Use "focus" to find $g(0)$.
- Use "focus" to find x where $g(x) = 0$.

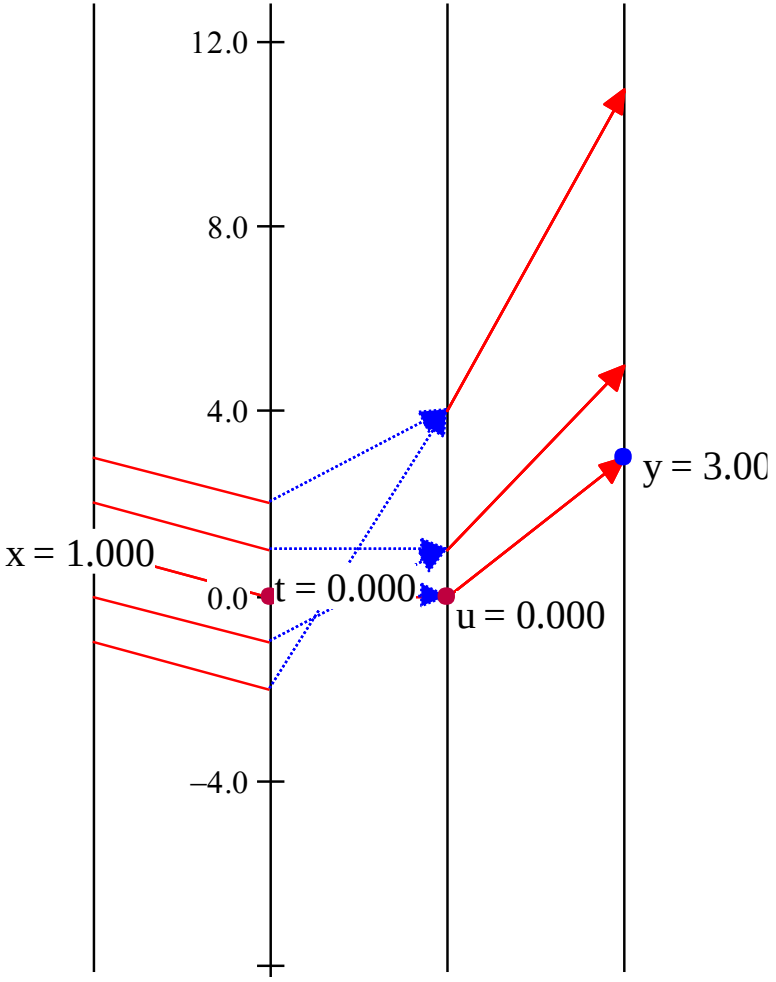
V. Examples: Composition and Inverses

Create Quadratic Example From Preface. ☺

$g(x) = 2\,(x-1)^2 + 3$

Steps for *g*:

1. Linear: Subtract 1.
2. Square result.
3. Linear: Multiply by 2 then add 3.
4. Visualize solving $g(x)=11$.



VI. Learning by Deconstruction.

A. Example: Henri Picciotto's Sampler.
   On-line version: Name That Function!
B. Example: Composition and Inverses.
C. Example: The Derivative
D. Other Topics : Choices on GeoGebra Tube

GeoGebraBook: Mapping Diagrams Take on Calculus and Complex Variables

http://ggbtu.be/bLGYKdQca

- Mapping Diagrams
- Linear Functions
- Limits and The Derivative
- 1st Derivative Analysis
- Diff'ls, D.Eq's and Euler's Method
- Integration and the Fundamental Theorem
- Taylor Polynomials
- Multivariable Functions

References

Basic Tools for GeoGebra (GG)

Resources from GeoGebra

Main Page for GeoGebra: http://geogebra.org/

Create your own on-line: http://web.geogebra.org/app/

Download Geogebra: http://www.geogebra.org/download

Help (wiki) pages: http://wiki.geogebra.org/en/Main_Page
http://wiki.geogebra.org/en/Tutorials

Mapping Diagrams and Functions

- Function Diagrams. by Henri Picciotto
  Excellent Resources!
  - Henri Picciotto's Math Education Page
  - Some rights reserved
- Flashman, Yanosko, Kim
  https://www.math.duke.edu//education/prep02/teams/prep-12/

Thanks

The End! REALLY!

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http://users.humboldt.edu/flashman