Making Sense of Solving Linear and Quadratic Equations with Mapping Diagrams

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- Mapping diagrams provide a valuable tool for visualizing functions and connects function concepts to solving equations in many contexts.
- In this presentation both linear and quadratic equations will be solved using mapping diagrams to make sense visually of the functions and steps used in common algebraic approaches to these problems.
- GeoGebra will be used as a dynamic tool to connect the concepts with technology.

Equations, Functions, and
Mapping Diagrams in Common Core
Links:

http://users.humboldt.edu/flashman/Presentations/CMC/CMC3.MD.LINKS.html

Mapping Diagram Sheets	**Mapping Diagram blanks (2 axis diagrams)**	**Mapping Diagram blanks (2 and 3 axes)**
Work/Spreadsheets	**Worksheet.pdf**	**Spreadsheet Template (Linear Functions)**
Section from MD from A B to C and DE (Drafts)	**Visualizing Functions: An Overview**	**Linear Functions** (LF) **Quadratic Functions(QF)**
GeoGebra	**Sketch to Visualize Solving a Linear Equation using Mapping Diagrams**	**Mapping Diagrams for Solving a Quadratic Equation**
YouTube Videos	**Using Mapping Diagrams to Visualize Linear Functions (10 Minutes)**	**Solving Linear Equations Visualized with Mapping Diagrams. (10 Minutes)**

Background Questions

- Are you familiar with Mapping Diagrams to visualize functions?
- Have you used Mapping Diagrams to teach functions?
- Have you used Mapping Diagrams to teach content besides function definitions?

Main Resource

- Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s. A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET FOR publication)
- http://users.humboldt.edu/flashman/MD/section-1.1VF.html

Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 1.a
  - Make tables for $m(x) = 2x$ and $s(x) = x+1$

x	m(x) =2x
2	
1	
0	
-1	
-2	

x	s(x) =x+1
2	
1	
0	
-1	
-2	

Mapping Diagram Prelim

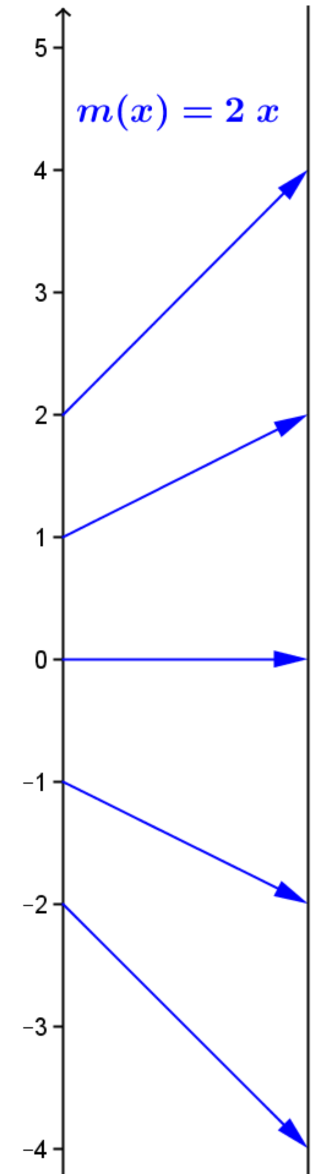
- Examples of mapping diagrams
  - Worksheet 1.b
  - On separate diagrams sketch mapping diagrams for $m(x) = 2x$ and $s(x)= x+1$

x	m(x) =2x
2	4
1	2
0	0
-1	-2
-2	-4

x	s(x) =x+1
2	3
1	2
0	1
-1	0
-2	-1

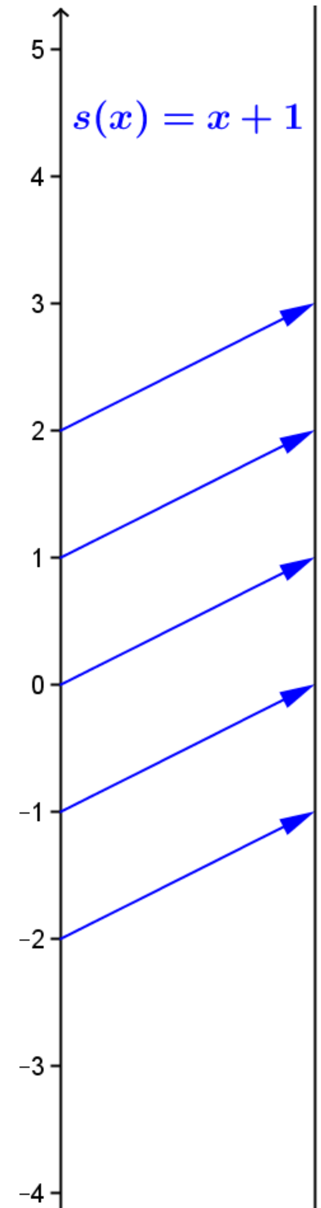
Worksheet 1.b Mapping Diagram:
m(x) = 2x

x	m(x) =2x
2	4
1	2
0	0
-1	-2
-2	-4



Worksheet 1.b Mapping Diagram:
s(x) = x+1

x	s(x)=x+1
2	3
1	2
0	1
-1	0
-2	-1



Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 2
  - a. First make table for $q(x) = x^2$.

x	$q(x) = x^2$
2	
1	
0	
-1	
-2	

Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 2
  - a. First make table for q.

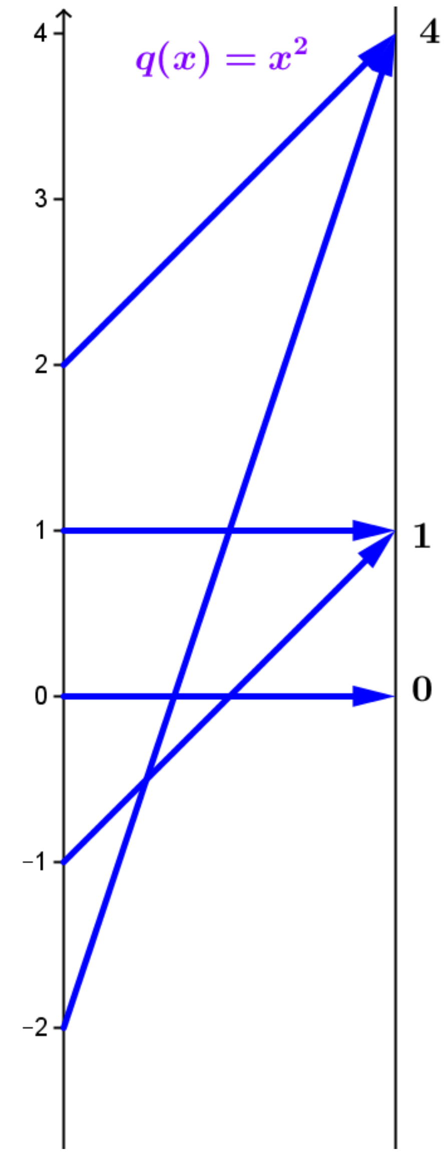
x	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4

  - b. Sketch a mapping diagram for $q(x) = x^2$.

Mapping Diagram Prelim

Worksheet 2.b. Mapping Diagram for $q(x) = x^2$

x	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4



Worksheet 3.a. Complete the following table for the composite function $f(x) = s(m(x)) = 2x + 1$

x	m(x)	f(x)=s(m(x))
2		
1		
0		
-1		
-2		

Worksheet 3.a.Complete the following table for the composite function $f(x) = s(m(x)) = 2x + 1$

x	m(x)	f(x)=s(m(x))
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3

Mapping Diagram Prelim

- Worksheet 3.b
- Use the table 3.a and the previous sketches of 1.b to draw a composite sketch of the mapping diagram with <u>3 axes for the composite function</u>

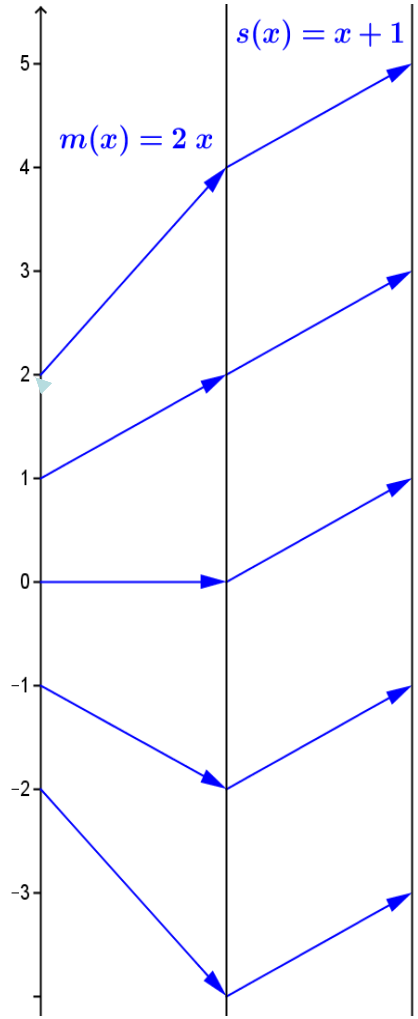
$$\underline{f(x) = h(g(x)) = 2x + 1}$$

Worksheet 3.b Draw a sketch for the mapping diagram with 3 axes of $f(x) = 2x + 1$.

x	m(x)	f(x)=s(m(x))
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3

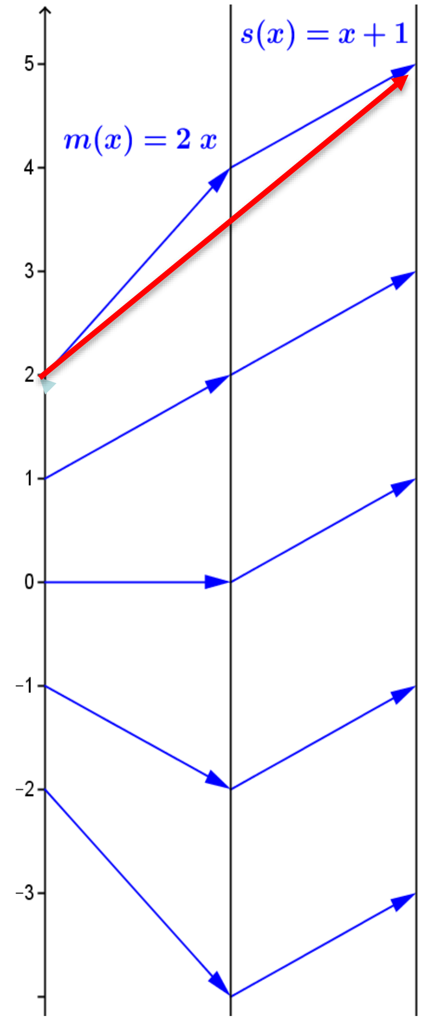
Worksheet 3.b Draw a sketch for the mapping diagram with <u>3 axes of $f(x) = 2x + 1$.</u>

x	m(x)	f(x)=s(m(x))
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3



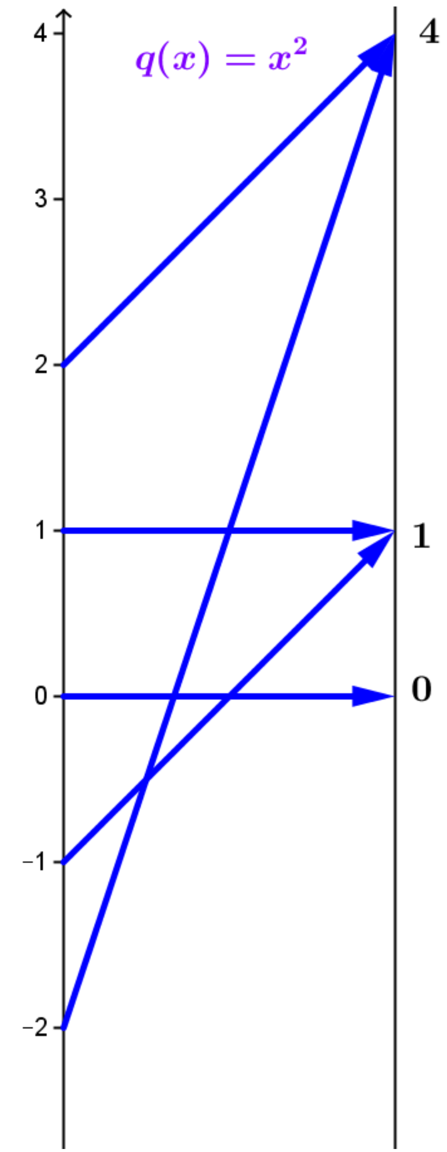
Worksheet 3.c Draw a sketch for the mapping diagram with 2 axes of $f(x) = 2x + 1$.

x	m(x)	f(x)=s(m(x))
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3



Worksheet 4 Mapping Diagram:
$q(x) = x^2$

x	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4



Worksheet 4.a

Complete the following tables for $q(x) = x^2$ and $R(x) = s(q(x)) = x^2 + 1$

x	q(x)	R(x)=s(q(x))
2		
1		
0		
-1		
-2		

Worksheet 4.a

Complete the following tables for $q(x) = x^2$ and $R(x) = s(q(x)) = x^2 + 1$

x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5

Worksheet 4.b

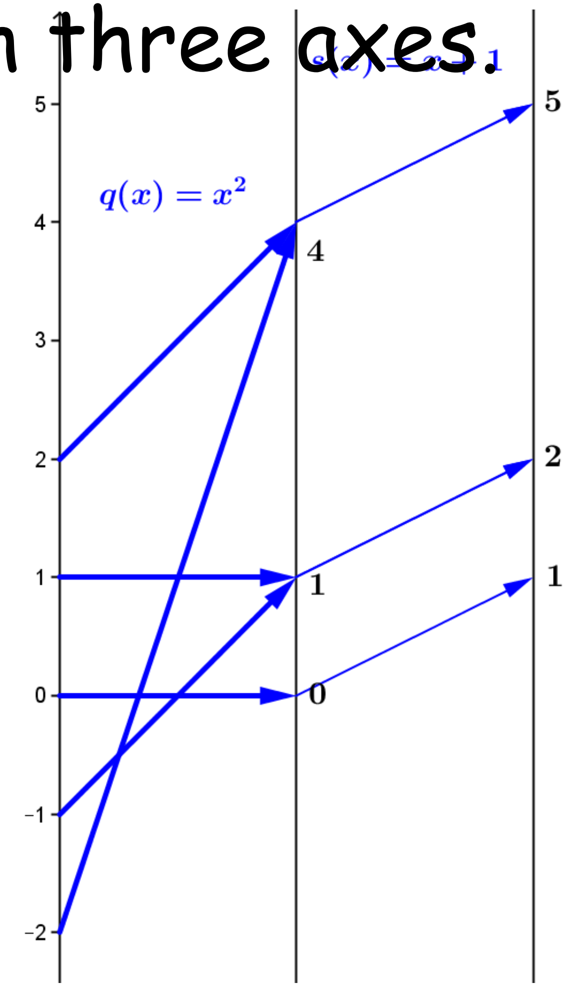
- 4.b Using the data from part a), sketch mapping diagrams for the composition $R(x) = s(q(x)) = x^2 + 1$ with three axes.

x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5

Worksheet 4.b

- 4.b Using the data from part a), sketch mapping diagrams for the composition $R(x) = s(q(x)) = x^2 + 1$ with three axes.

x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5



Worksheet 4.b

- 4.b Using the data from part a), sketch mapping diagrams for the composition $R(x) = s(q(x)) = x^2 + 1$ with two axes.

x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5

An Old Friend: Solving A Linear Equation

- Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

Find x.

An Old Friend:
Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$
\begin{array}{rcl}
2x + 1 & = & 5 \\
-1 & = & -1 \\
\hline
2x & = & 4
\end{array}
$$

An Old Friend: Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

$$\underline{\quad -1 = -1}$$

$$2x = 4$$

$$\underline{1/2(2x) = 1/2(4)}$$

$$x = 2$$

An Old Friend: Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

$$-1 = -1$$

$$2x = 4$$

$$1/2(2x) = 1/2(4)$$

$$x = 2$$

Check!

$$2x+1 = 2*2 + 1 \stackrel{!}{=} 5$$

Linear Equations Use Linear Functions!

Linear Equations

$$2x + 1 = 5$$
$$-1 = -1$$
$$2x = 4$$
$$1/2(2x) = 1/2(4)$$
$$x = 2$$

Check:

$2x + 1 = 2*2 + 1 = 5$

Linear Functions

$f(x) = 2x + 1$

Linear Equations Use Linear Functions!

Linear Equations

$$2x + 1 = 5$$

$$-1 = -1$$

$$2x = 4$$

$$1/2(2x) = 1/2(4)$$

$$x = 2$$

Check:

$2x + 1 = 2*2 + 1 = 5$

Linear Functions

$f(x) = 2x + 1$

$m(x) = 2x;\ s(x) = x + 1$

$f(x) = s(m(x))$

Worksheet 5.b Solving 2x + 1 = 5 visualized with a mapping diagram

Algebra:

2x + 1 = 5

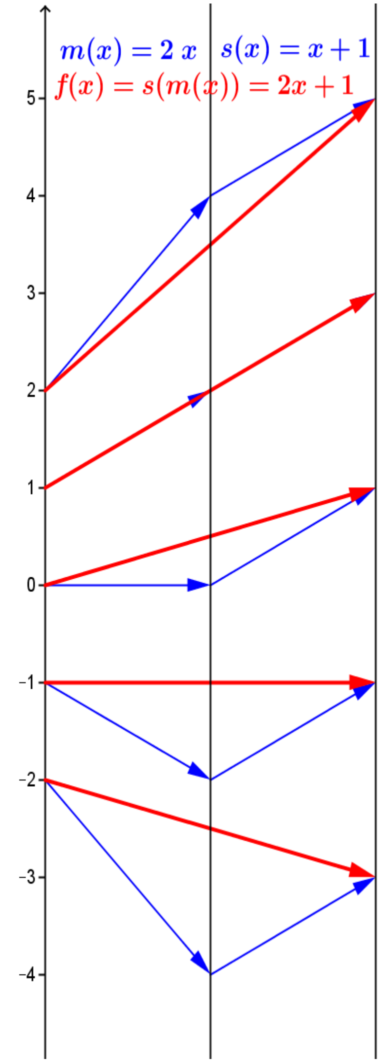
-1 = -1

2x = 4

1/2(2x) =1/2(4)

x = 2

How does the MD for the function VISUALIZE the algebra?



Worksheet 5.b Solving 2x + 1 = 5 visualized with a mapping diagram

Algebra:

$2x + 1 = 5$

$\quad\;\; -1 = -1$

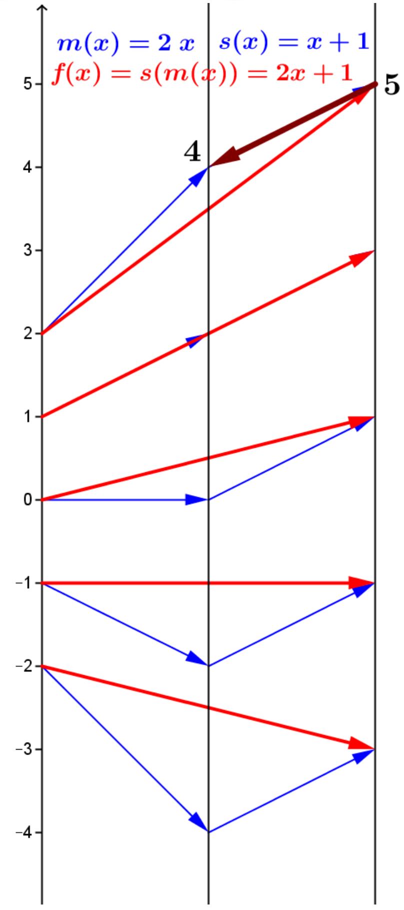
$2x \quad\; = 4$

Function:

$f(x)=s(m(x)) = 5$

"Undo s"

$m(x) = 4$



Worksheet 5.b Solving 2x + 1 = 5 visualized with a mapping diagram

Algebra:

2x + 1 = 5

-1 = -1

2x = 4

1/2(2x) =1/2(4)

x = 2

Function:

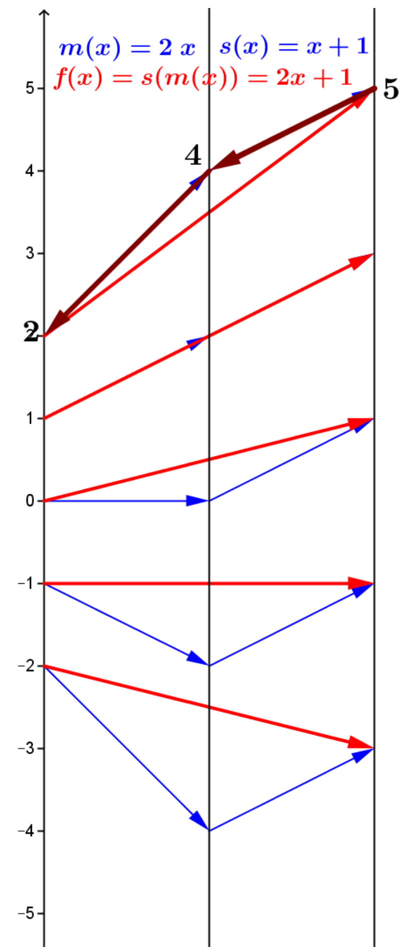
f(x)=s(m(x)) = 5

“Undo s”

m(x) = 4

“Undo m”

x = 2



Worksheet 5.b Solving 2x + 1 = 5 visualized with a mapping diagram

Algebra:

$2x + 1 = 5$

$\underline{\quad -1 = -1}$

$2x = 4$

$\underline{1/2(2x) = 1/2(4)}$

$x = 2$

Function:

$f(x)=s(m(x)) = 5$

"Undo s"

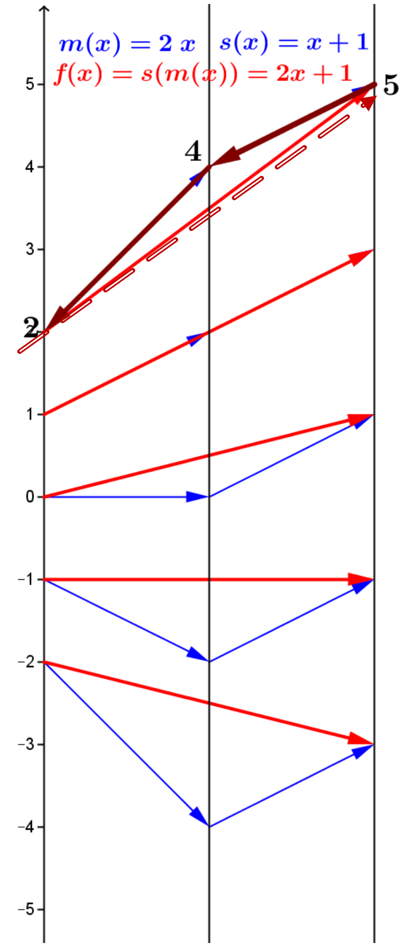
$m(x) = 4$

"Undo m"

$x = 2$

CHECK! ☺

$f(2)\overset{!}{=}5$



Worksheet 5.b Solving 2x + 1 = 5 visualized on GeoGebra

Algebra:

$2x + 1 = 5$

$-1 = -1$

$2x = 4$

$1/2(2x) = 1/2(4)$

$x = 2$

Function:

$f(x)=s(m(x)) = 5$

“Undo s”

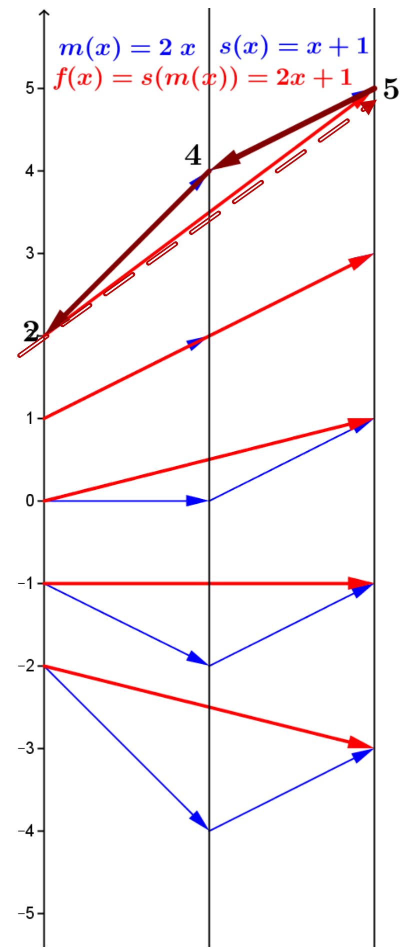
$m(x) = 4$

“Undo m”

$x = 2$

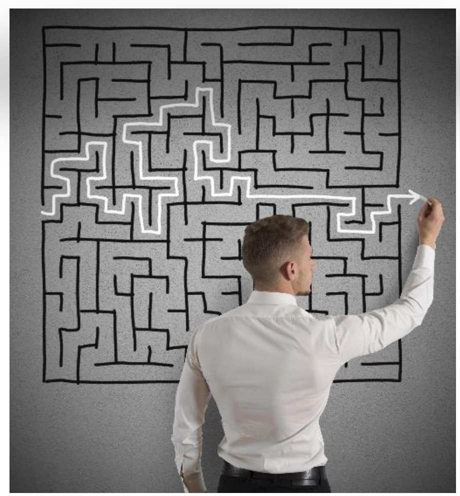
CHECK! ☺

$f(2) \overset{!}{=} 5$



Challenge: Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

The Great Mind Challenge
Initiate Collaborate Innovate



Worksheet 6.a Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Understand the problem

- $2(x-3)^2 + 1$ is a function of x.
    - $P(x) = 2(x-3)^2 + 1$
- Find any and all x where $P(x) = 9$.
- $2(x-3)^2 + 1$ is a composition of functions
    - $P(x) = s(m(q(z(x))))$ where
    - $z(x) =$
    - $q(x) =$
    - $m(x) =$
    - $s(x) =$

Worksheet 6.a Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Understand the problem

- $2(x-3)^2 + 1$ is a function of x.
  - $P(x) = 2(x-3)^2 + 1$
- Find any and all x where $P(x) = 9$.
- $2(x-3)^2 + 1$ is a composition of functions
  - $P(x) = s(m(q(z(x))))$ where
  - $z(x) = x-3$;
  - $q(x) = x^2$;
  - $m(x) = 2x$;
  - $s(x) = x+1$.

Worksheet 6.a Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram.

Make a plan

- Find any and all x where $P(x) = 9$.
- Construct mapping diagram for P as a composition of function :

$$P(x) = s(m(q(z(x))))$$

- Undo $P(x) = 9$ by undoing each step of P
  - Undo $s(x) = x+1$
  - Undo $m(x) = 2x$
  - Undo $q(x) = x^2$
  - Undo $z(x) = x-3$
- Check results to see that $P(x) = 9$

Worksheet 6.b Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram.

Execute the **plan**

- Construct mapping diagram for P as a composition of function :

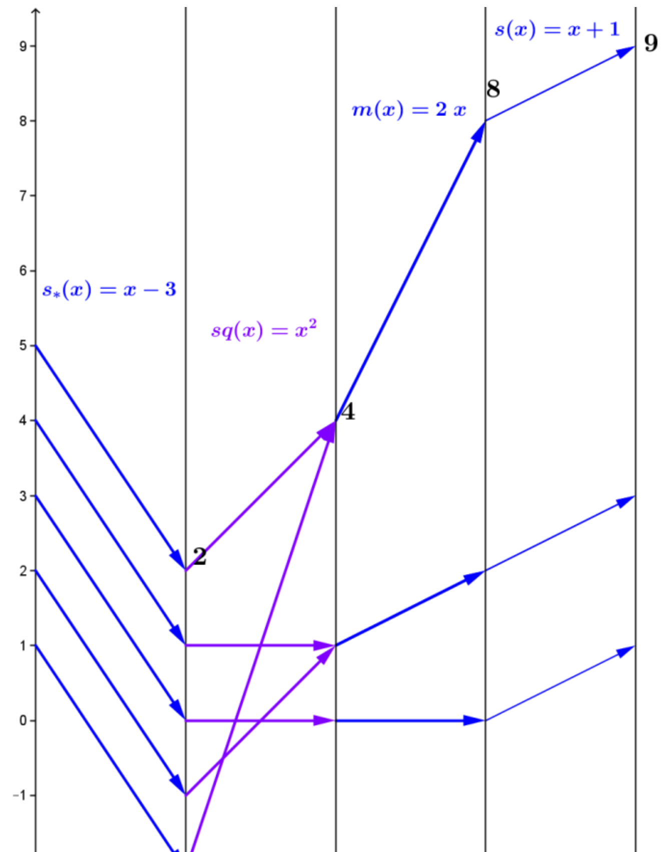
$$P(x) = s(m(q(z(x))))$$

Worksheet 6.b Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram.

Execute the **plan**

- Construct mapping diagram for P as a composition of function :

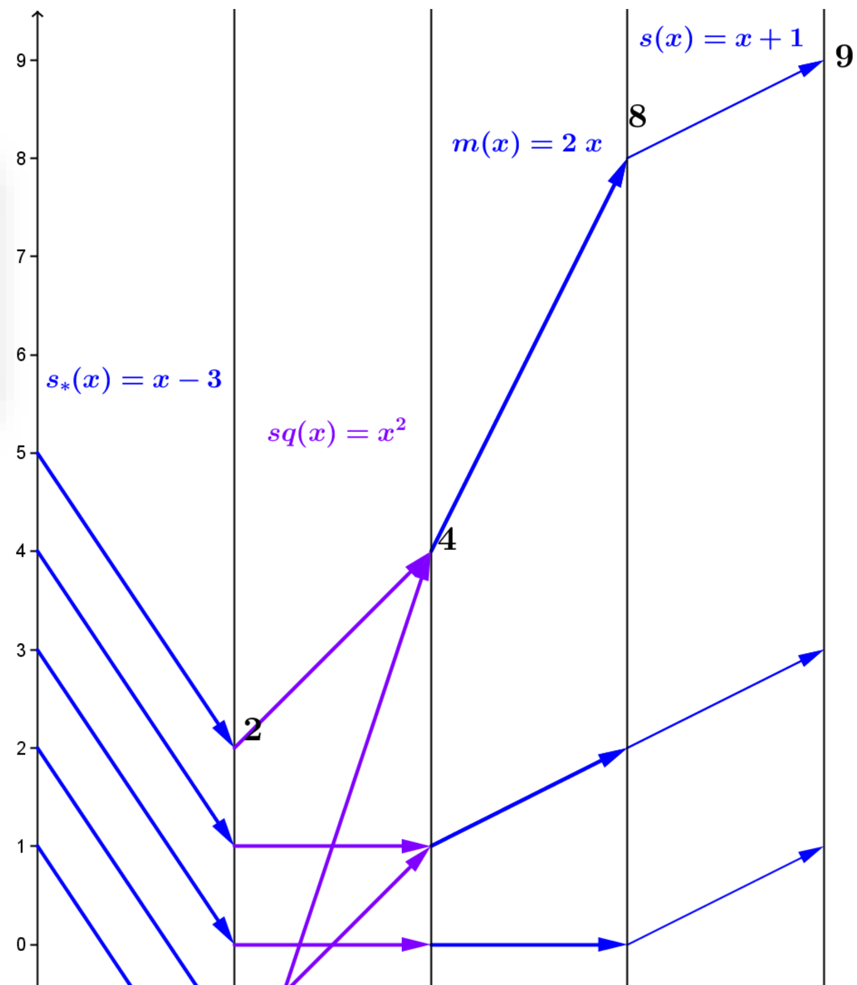
$$P(x) = s(m(q(z(x))))$$



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Execute the plan

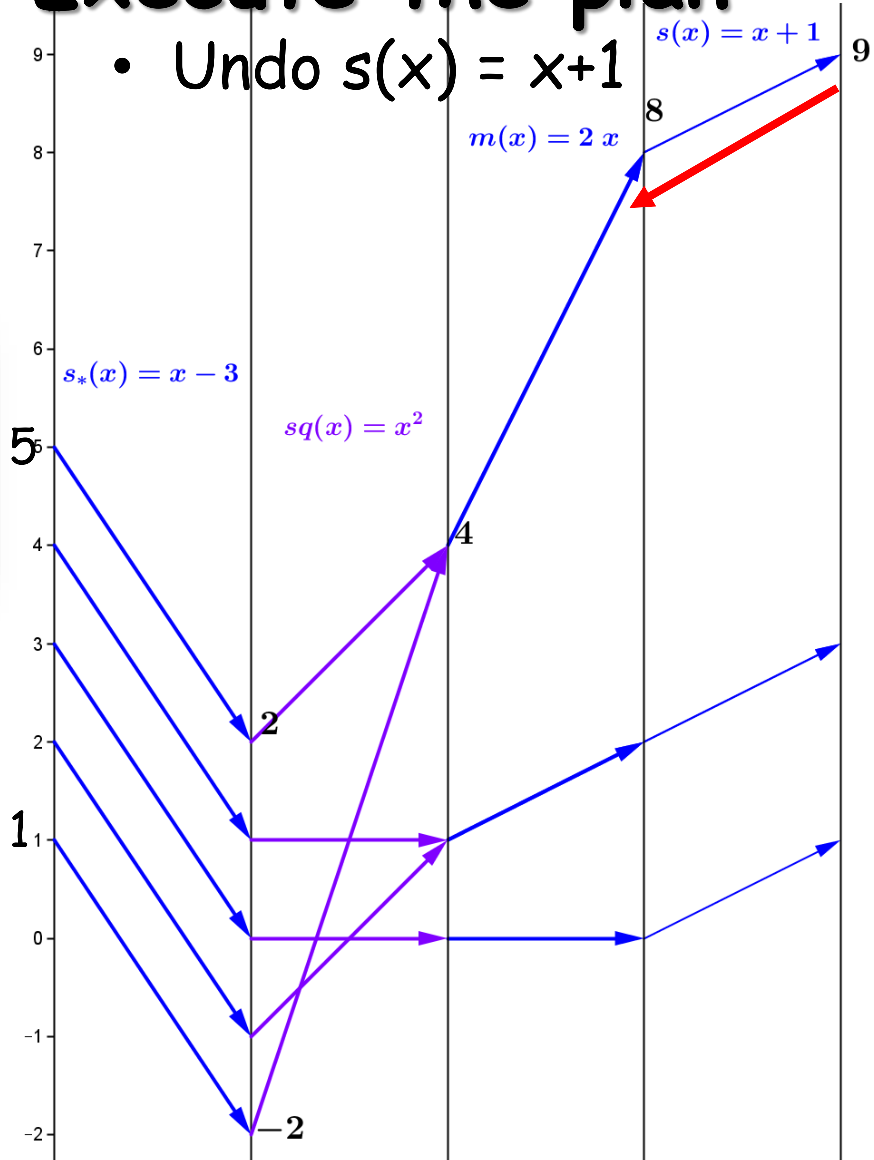
- Find any and all x where P(x) = 9.



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Execute the plan

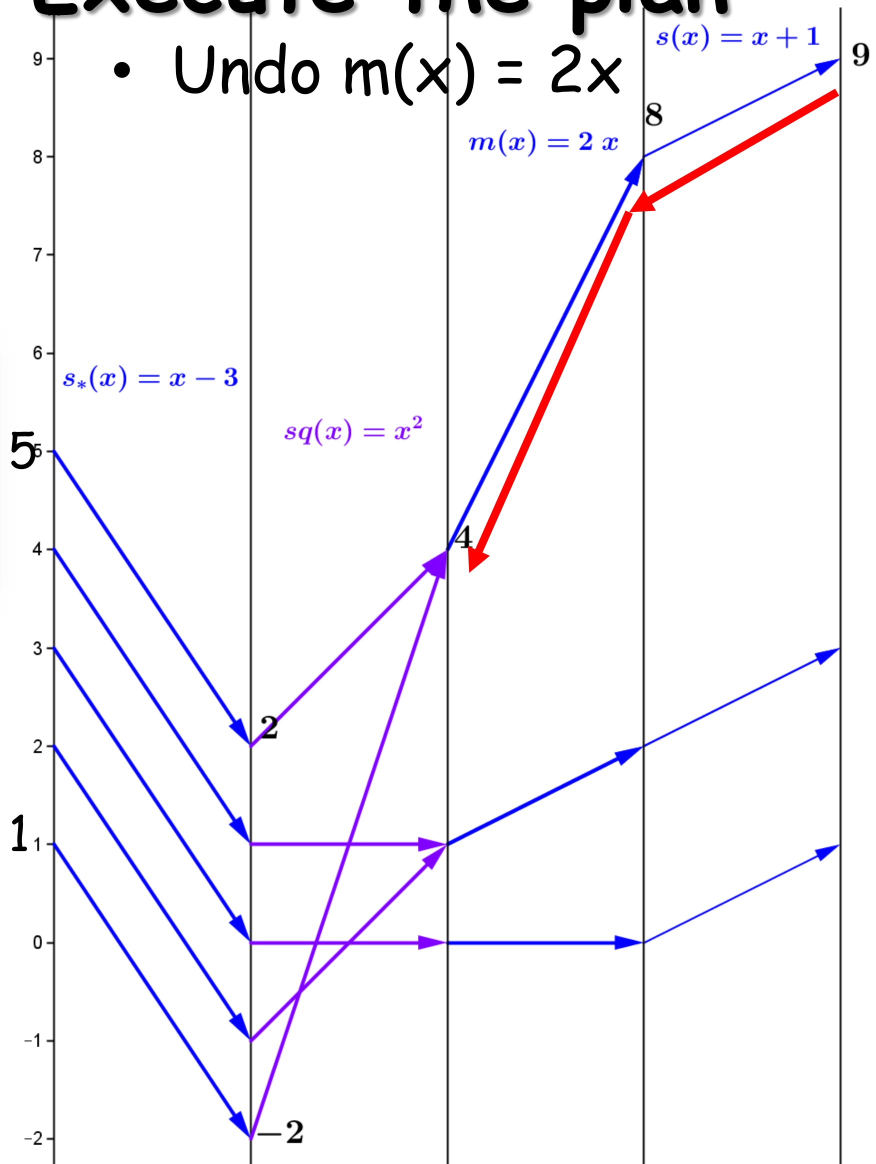
- Undo $s(x) = x+1$



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Execute the plan

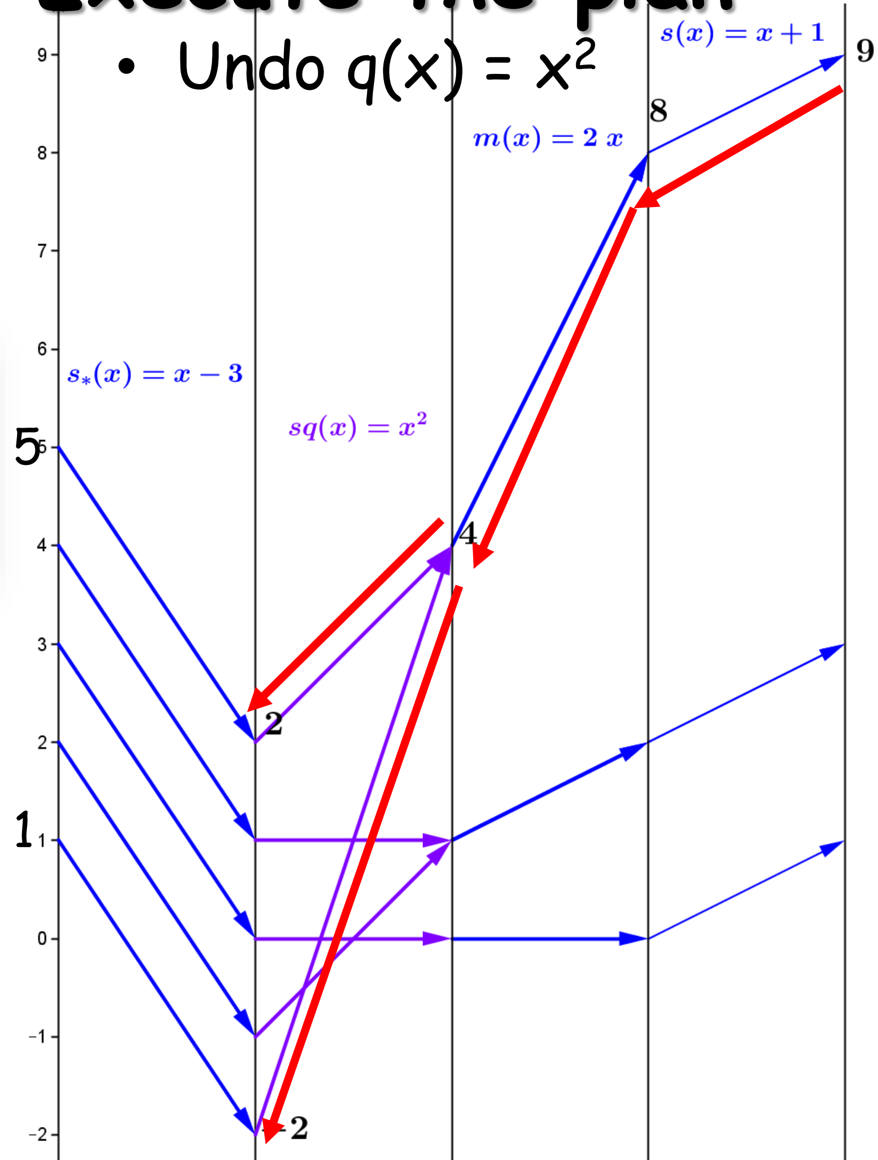
- Undo $m(x) = 2x$



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Execute the plan

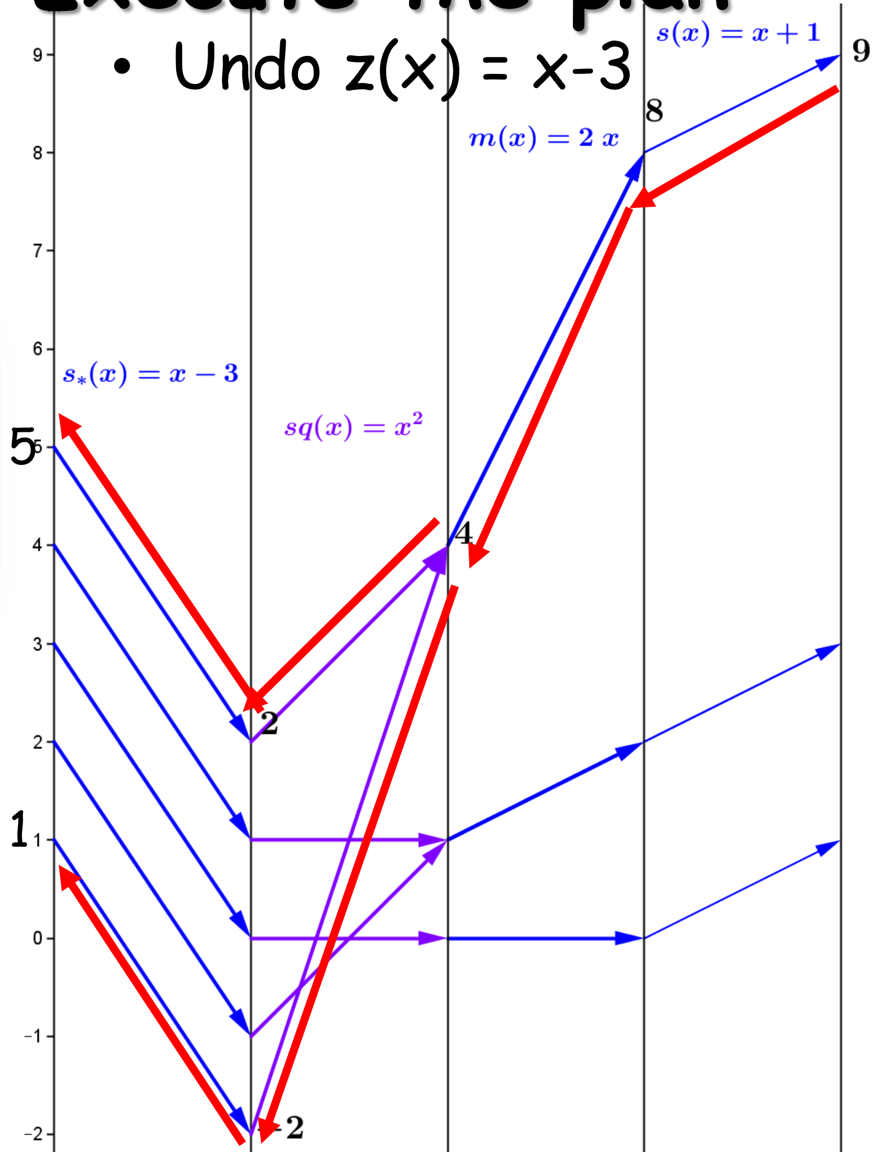
- Undo $q(x) = x^2$



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

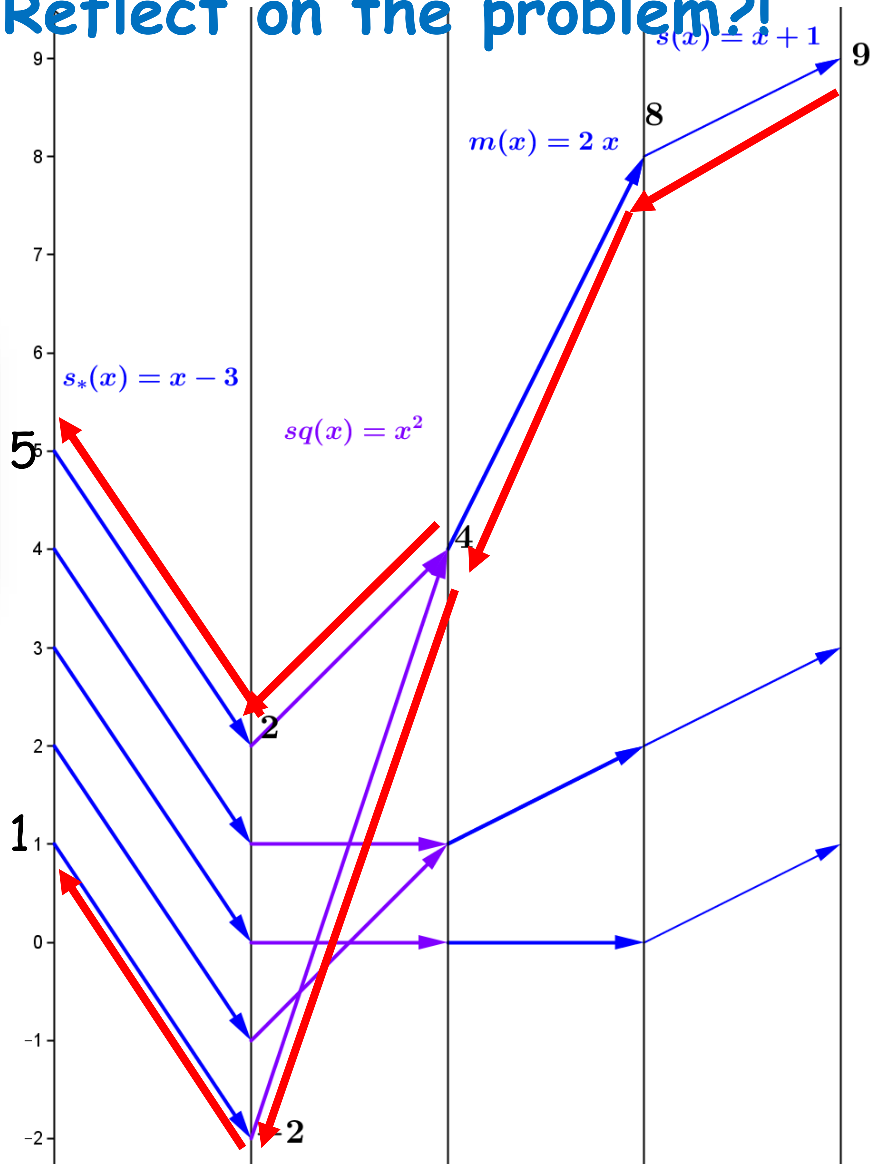
Execute the plan

- Undo $z(x) = x-3$



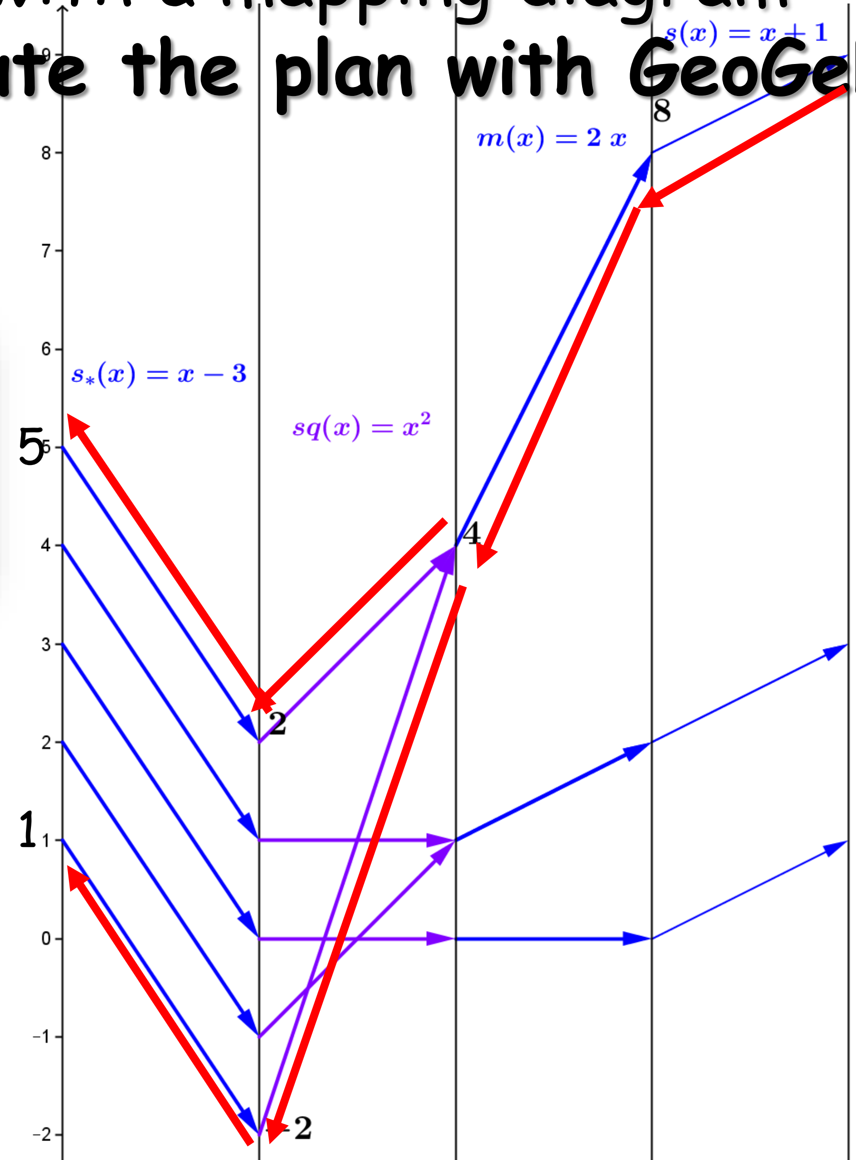
Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Reflect on the problem?!



Challenge: Worksheet 6.c
Solve $2(x-3)^2 + 1 = 9$
with a mapping diagram

Execute the plan with GeoGebra



Technology Examples

- Excel examples
- Geogebra examples

Overtime?
Simple Examples are important!

- $f(x) = x + C$ Added value: C
- $f(x) = mx$ Scalar Multiple: m

  Interpretations of m:
  - slope
  - rate
  - Magnification factor
  - $m > 0$: Increasing function
  - $m < 0$: Decreasing function
  - $m = 0$: Constant function

Simple Examples are important!

$f(x) = mx + b$ with a mapping diagram --
Five examples:
Back to Worksheet Problem #7

- Example 1: $m = -2$; $b = 1$: $f(x) = -2x + 1$
- Example 2: $m = 2$; $b = 1$: $f(x) = 2x + 1$
- Example 3: $m = \frac{1}{2}$; $b = 1$: $f(x) = \frac{1}{2}x + 1$
- Example 4: $m = 0$; $b = 1$: $f(x) = 0x + 1$
- Example 5: $m = 1$; $b = 1$: $f(x) = x + 1$

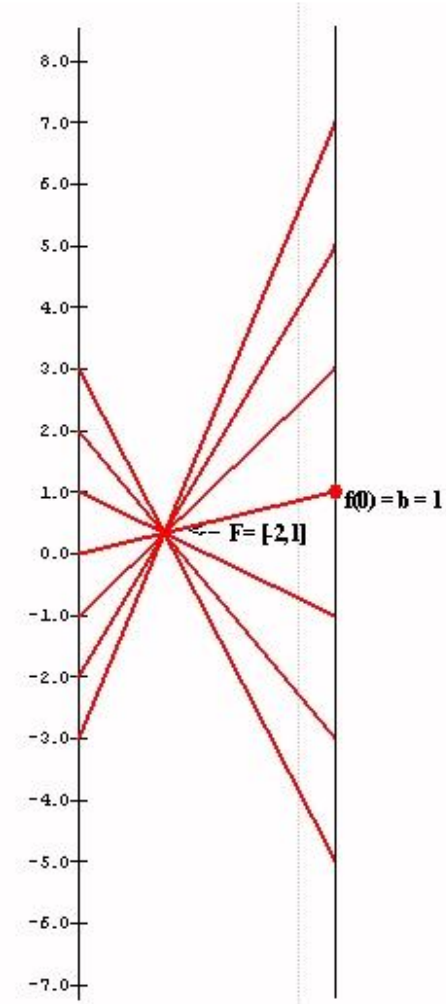
Visualizing *f* (*x*) = ***mx*** + ***b*** with a mapping diagram -- Five examples:

Example 1: m = -2; b = 1

f (x) = **-2**x + **1**

- Each arrow passes through a single point, which is labeled **F = [- 2,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, -2*x* + 1,

    which corresponds to the linear function's value for the point/number, *x*.



Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

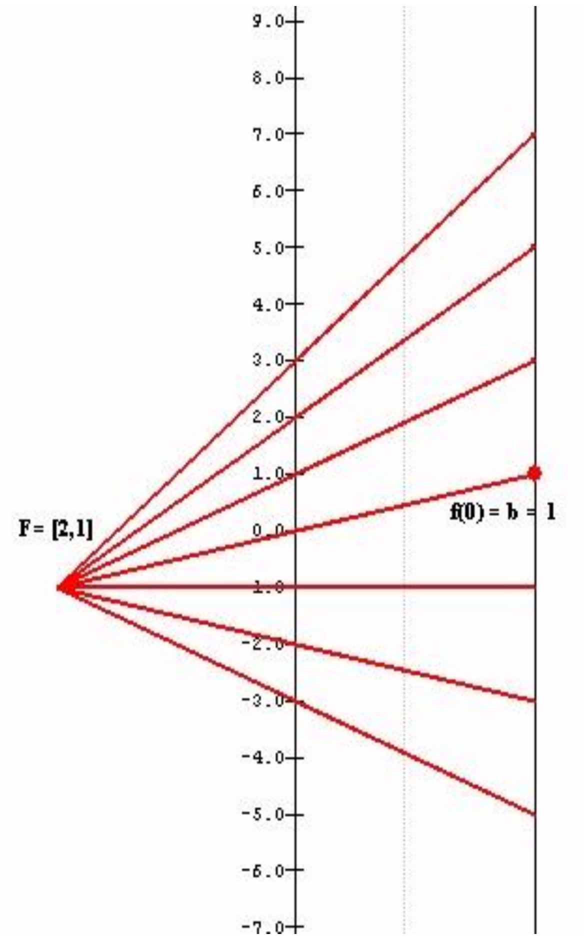
Example 2: $\boldsymbol{m} = 2$; $\boldsymbol{b} = 1$
$f(x) = 2x + 1$

Each arrow passes through a single point, which is labeled

F = [2,1].

- The point **F** completely determines the function f.
  - **given a point** / number, x, on the source line,
  - there is a **unique arrow passing through F**
  - **meeting** the target line at **a unique point** / number, $2x + 1$,

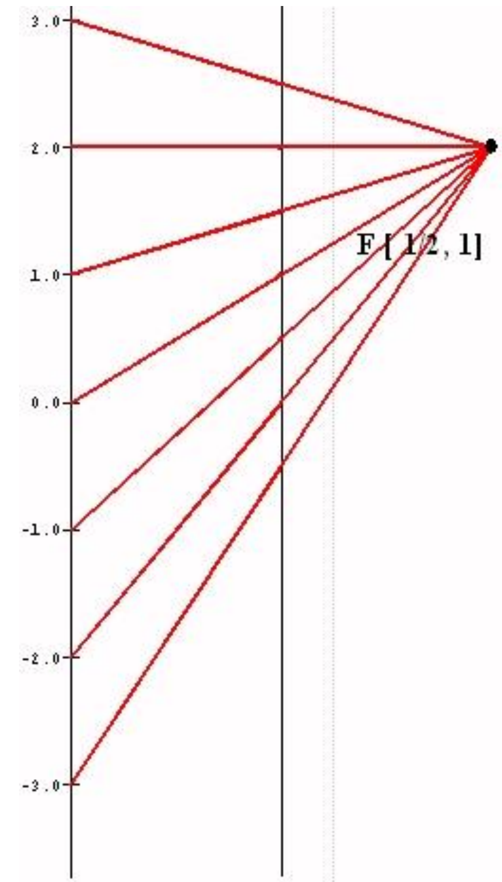
which corresponds to the linear function's value for the point/number, x.



Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

- **Example 3: m = 1/2; b = 1**
  $f(x) = \frac{1}{2}x + 1$
- Each arrow passes through a single point, which is labeled **F = [1/2,1]**.
  - The point **F** completely determines the function f.
    - **given a point** / number, x, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $\frac{1}{2}x + 1$,

    which corresponds to the linear function's value for the point/number, x.

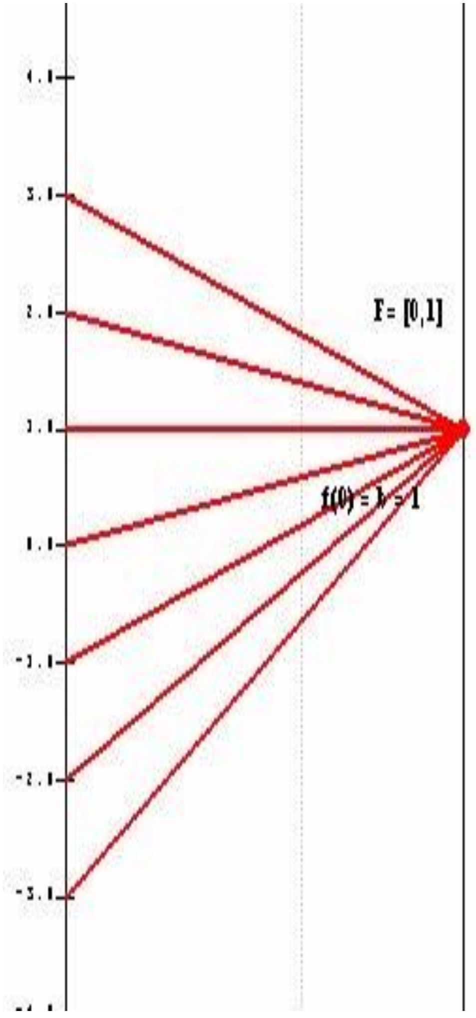


Visualizing $f(x) = \boldsymbol{mx + b}$ with a mapping diagram -- Five examples:

- **Example 4: $m = 0$; $b = 1$**
  $f(x) = 0\ x + 1$

- Each arrow passes through a single point, which is labeled **F = [0,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $f(x)=1$,

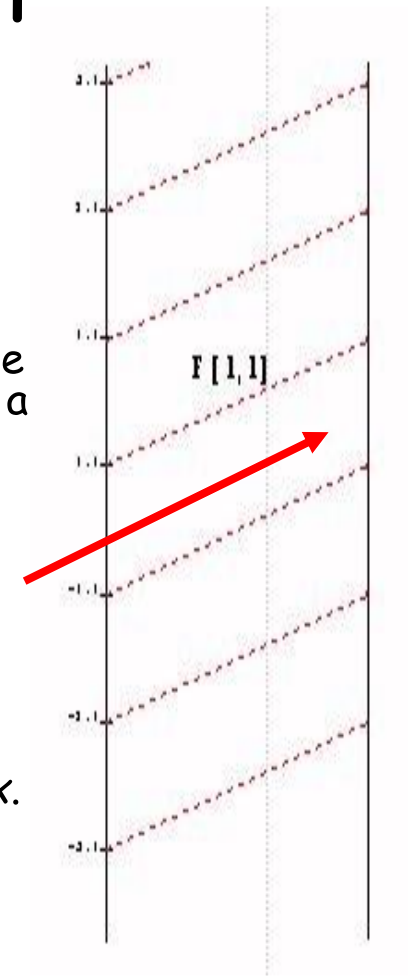
    which corresponds to the linear function's value for the point/number, *x*.



Visualizing $f(x) = \boldsymbol{mx} + \boldsymbol{b}$ with a mapping diagram -- Five examples

Example 5: $m = 1$; $b = 1$

$f(x) = x + 1$



- Unlike the previous examples, in this case it is not a single point that determines the mapping diagram, but the single arrow from 0 to 1, which we designate as F[1,1]
- It can also be shown that this single arrow completely determines the function. Thus, given a point / number, x, on the source line, there is a unique arrow passing through ***x* parallel to** F[1,1] meeting the target line a unique point / number, $x + 1$, which corresponds to the linear function's value for the point/number, x.
  - The single arrow completely determines the function f.
    - **given a point** / number, x, on the source line,
    - there is a **unique arrow** through ***x* parallel to** F[1,1]
    - **meeting** the target line at **a unique point** / number, $x + 1$,

which corresponds to the linear function's value for the point/number, x.

Simple Examples are important!

- $f(x) = x + C$ Added value: C
- $f(x) = mx$ Scalar Multiple: m

  Interpretations of m:
  - slope
  - rate
  - Magnification factor
  - $m > 0$: Increasing function
  - $m < 0$: Decreasing function
  - $m = 0$: Constant function

Function-Equation Questions

with linear focus points (Problem 8.a)

- Use a focus point in the mapping diagram to solve a linear equation:

$$2x+1 = 5$$

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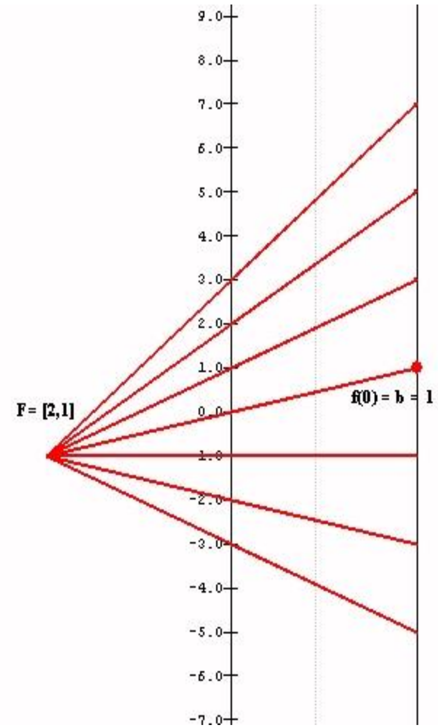

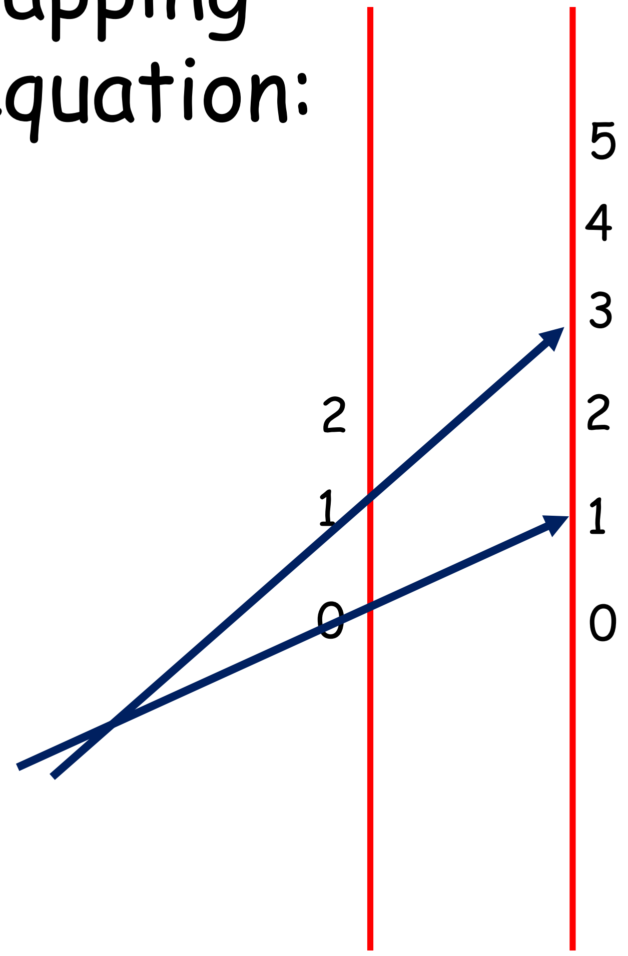

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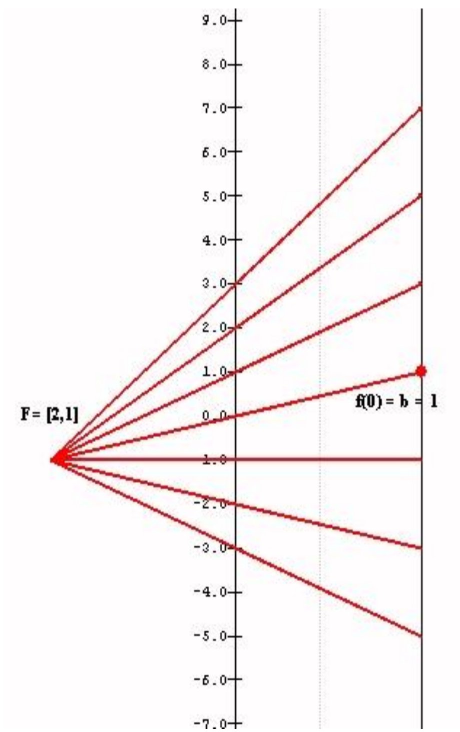

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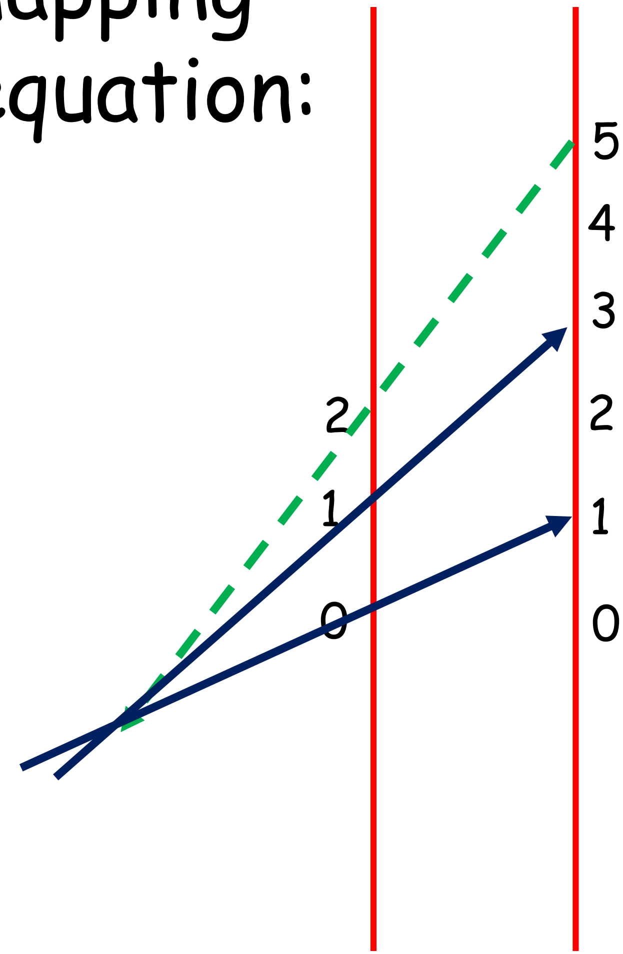

Function-Equation Questions

with linear focus points (Problem 8)

Suppose f is a linear function with $f(1) = 3$ and $f(3) = -1$.

Without algebra

- 8.b Use a focus point to find $f(0)$.
- 8.c Use a focus point to find x where $f(x) = 0$.

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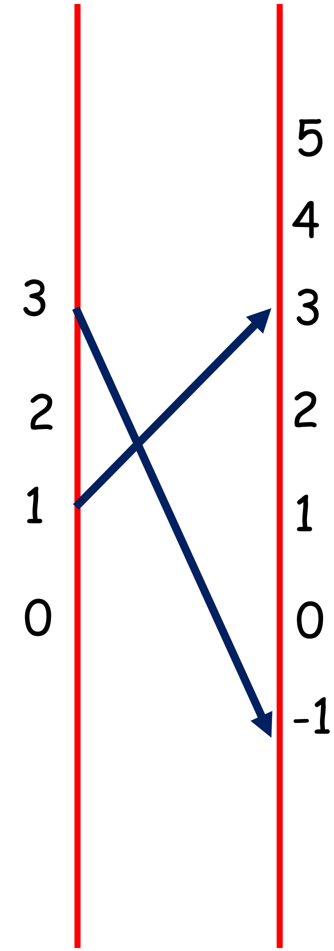

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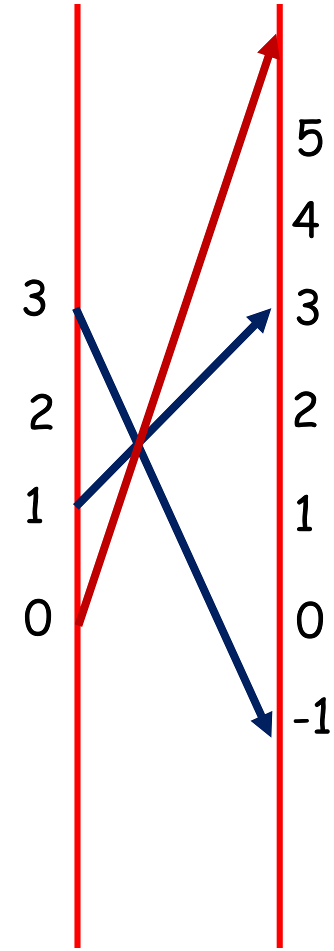

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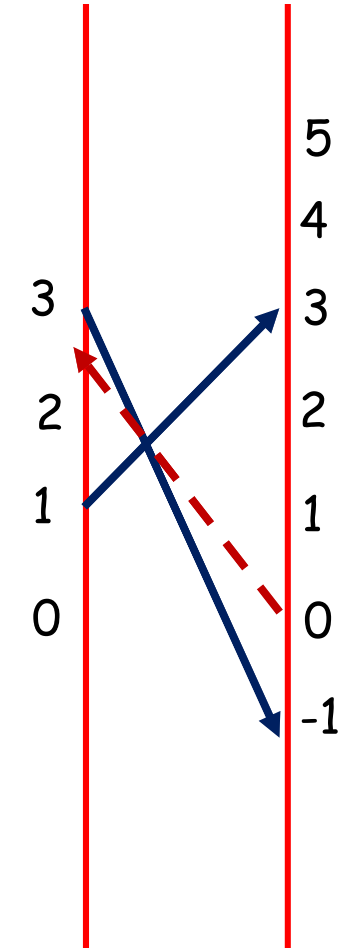

Thanks

The End!

☺

Questions?

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References

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- Function Diagrams. **by Henri Picciotto Excellent Resources!**
  - Henri Picciotto's Math Education Page
  -

- Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s. A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET FOR publication) **http://users.humboldt.edu/flashman/MD/section-1.1VF.html**

- **Mapping Diagrams and Graphs…** Visualizing linear functions using mapping diagrams and graphs. **tube.geogebra.org** Martin Flashman

Thanks

The End!   REALLY!

☺

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