Equations, Functions, and Mapping Diagrams in Common Core

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- The Common Core emphasizes making sense of solving equations and using functions.
- Mapping diagrams provide a valuable tool for visualizing functions and connects function concepts to solving equations in many contexts.
- In this presentation both linear and quadratic equations will be solved using mapping diagrams to make sense visually of the functions and steps used in common algebraic approaches to these problems.
- GeoGebra will be used as a dynamic tool to connect the concepts with technology.

Equations, Functions, and Mapping Diagrams in Common Core Links:

http://users.humboldt.edu/flashman/Presentations/CMC/CMC.MD.LINKS.html

Mapping Diagram Sheets	**Mapping Diagram blanks (2 axis diagrams)**	**Mapping Diagram blanks (2 and 3 axes)**
Work/Spreadsheets	**Worksheet.pdf**	**Spreadsheet Template (Linear Functions)**
Section from MD from A B to C and DE (Drafts)	**Visualizing Functions: An Overview**	**Linear Functions** (LF) **Quadratic Functions(QF)**
GeoGebra	**Sketch to Visualize Solving a Linear Equation using Mapping Diagrams**	**Mapping Diagrams for Solving a Quadratic Equation**
YouTube Videos	**Using Mapping Diagrams to Visualize Linear Functions (10 Minutes)**	**Solving Linear Equations Visualized with Mapping Diagrams. (10 Minutes)**

The Two Most Important Mathematical Concepts!

Number **Function**

Common Core Connections (Grade 8)

- Define, evaluate, and compare functions.
  - **Understand that a function is a rule that assigns to each input exactly one output.** The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
  - **Compare properties of two functions each represented in a different way** (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function …. determine which function has the greater rate of change.
  - **Interpret the equation y = mx + b as defining a linear function**, whose graph is a straight line; give examples of functions that are not linear.

Common Core Connections (Grade 8)

- Use functions to model relationships between quantities..
  - Construct a function to model a linear relationship between two quantities. **Determine the rate of change and initial value of the function** from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. **Interpret the rate of change and initial value of a linear function** in terms of the situation it models, and in terms of its graph or a table of values.
  - **Describe qualitatively the functional relationship between two quantities** by analyzing a graph (e.g., **where the function is increasing or decreasing, linear or nonlinear**).

Common Core Connections (Functions)

Functions Overview

Interpreting Functions

Understand the concept of a function and use function notation

Interpret functions that arise in applications in terms of the context

Analyze functions using different representations

Building Functions

Build a function that models a relationship between two quantities

Build new functions from existing functions

Linear, Quadratic, and Exponential Models

Construct and compare linear and exponential models and solve problems

Interpret expressions for functions in terms of the situation they model.

Common Core Connections

(Functions)

Mathematical Practices

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- **Construct viable arguments** and critique the reasoning of others.
- Model with mathematics.
- **Use appropriate tools strategically.**
- Attend to precision.
- **Look for and make use of structure.**
- **Look for and express regularity in repeated reasoning.**

Background Questions

- Are you familiar with Mapping Diagrams to visualize functions?
- Have you used Mapping Diagrams to teach functions?
- Have you used Mapping Diagrams to teach content besides function definitions?

Visualizing Linear Functions

- Linear functions are a basic element of the Common Core.
- There is a sensible way to visualize them using "mapping diagrams."
- Examples of <u>important Common Core function features (like monotonicity and intercepts)</u> can be illustrated sensibly with mapping diagrams.
- Activities for students using mapping diagrams engage understanding <u>for many function concepts</u>.
- Creating Mapping diagrams can use simple tools (straight edges) as well as technology.

Visualizing Quadratic Functions

- Quadratic functions are another basic element of the Common Core.
- There is a sensible way to visualize them using "mapping diagrams."
- Examples of <u>important Common Core function features (like extremes and intercepts)</u> can be illustrated sensibly with mapping diagrams.
- Activities for students using mapping diagrams engage understanding <u>for function concepts</u>.
- Creating Mapping diagrams can use simple tools (straight edges) as well as technology.

Main Resource

- Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s. A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET FOR publication)

- http://users.humboldt.edu/flashman/MD/section-1.1VF.html

Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 1.a
  - Make tables for $m(x) = 2x$ and $s(x) = x+1$

x	m(x) =2x
2	
1	
0	
-1	
-2	

x	s(x) =x+1
2	
1	
0	
-1	
-2	

Mapping Diagram Prelim

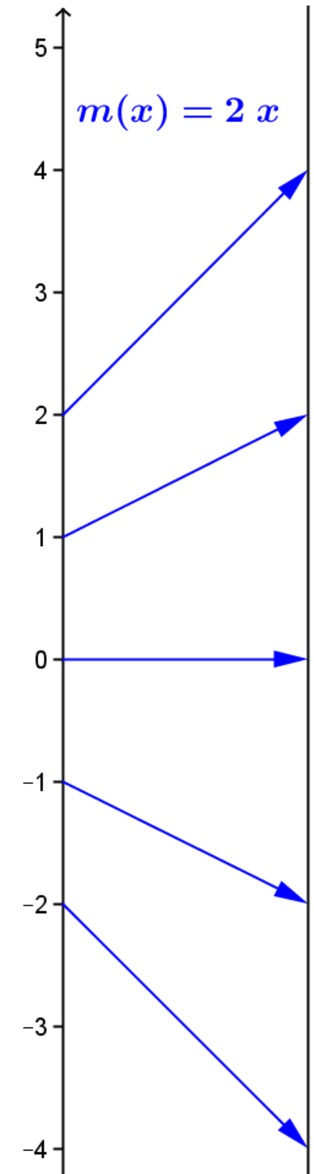
- Examples of mapping diagrams
  - Worksheet 1.b
  - On separate diagrams sketch mapping diagrams for $m(x) = 2x$ and $s(x)= x+1$

x	m(x) =2x
2	4
1	2
0	0
-1	-2
-2	-4

x	s(x) =x+1
2	3
1	2
0	1
-1	0
-2	-1

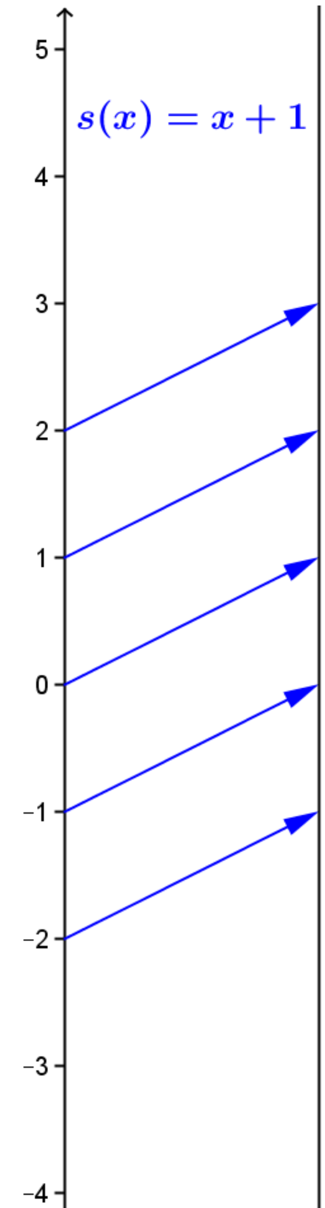
Worksheet 1.b Mapping Diagram: $m(x) = 2x$

x	m(x) =2x
2	4
1	2
0	0
-1	-2
-2	-4



Worksheet 1.b Mapping Diagram:
s(x) = x+1

x	s(x)=x+1
2	3
1	2
0	1
-1	0
-2	-1



Mapping Diagram Prelim

- Examples of mapping diagrams
  - Worksheet 2
  - a. First make table for $q(x) = x^2$.

x	$q(x) = x^2$
2	
1	
0	
-1	
-2	

Mapping Diagram Prelim

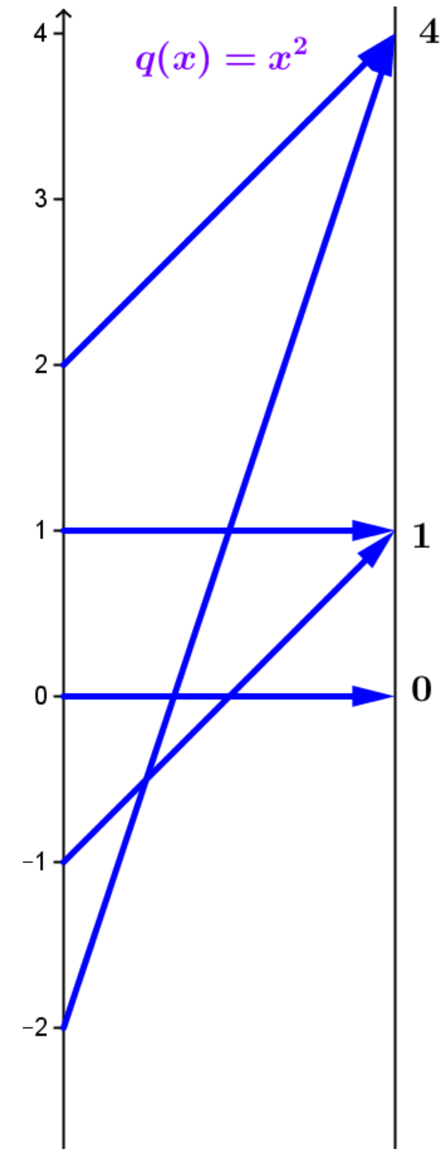
- Examples of mapping diagrams
  - Worksheet 2
  - a. First make table for q.

x	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4

  - b. Sketch a mapping diagram for $q(x) = x^2$.

Mapping Diagram Prelim
Worksheet 2.b. Mapping Diagram for $q(x) = x^2$

x	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4



Worksheet 3.a.Complete the following table for the composite function $f(x) = s(m(x)) = 2x + 1$

x	m(x)	f(x)=s(m(x))
2		
1		
0		
-1		
-2		

Worksheet 3.a.Complete the following table for the composite function $f(x) = s(m(x)) = 2x + 1$

x	m(x)	f(x)=s(m(x))
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3

Mapping Diagram Prelim

- Worksheet 3.b
- Use the table 3.a and the previous sketches of 1.b to draw a composite sketch of the mapping diagram with <u>3 axes for the composite function</u>

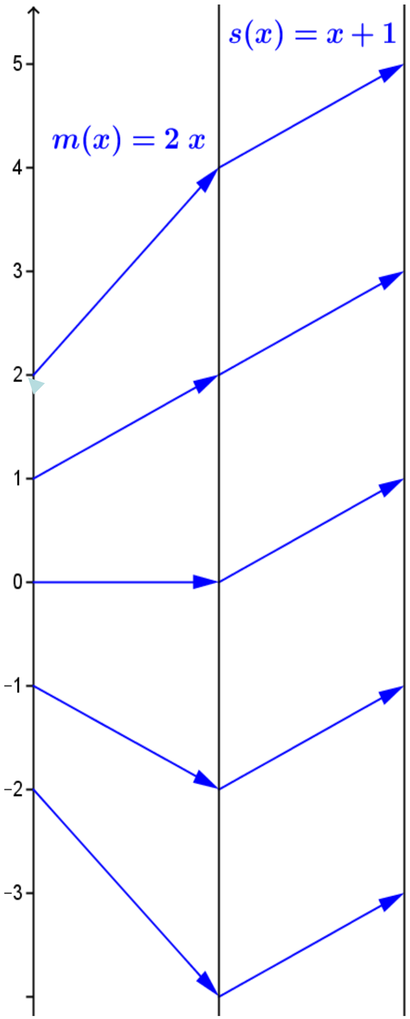
  <u>$f(x) = h(g(x)) = 2x + 1$</u>

Worksheet 3.b Draw a sketch for the mapping diagram with 3 axes of $f(x) = 2x + 1$.

x	m(x)	f(x)=s(m(x))
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3

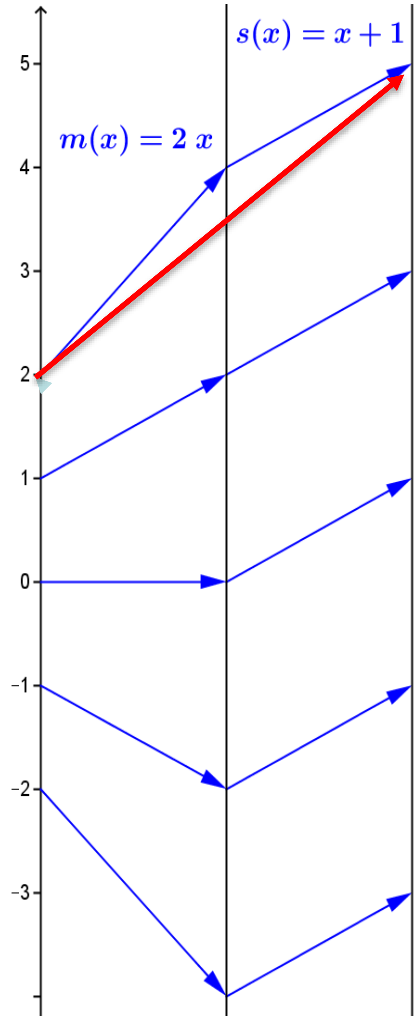
Worksheet 3.b Draw a sketch for the mapping diagram with <u>3 axes of $f(x) = 2x + 1$.</u>

x	m(x)	f(x)=s(m(x))
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3



Worksheet 3.c Draw a sketch for the mapping diagram with 2 axes of $f(x) = 2x + 1$.

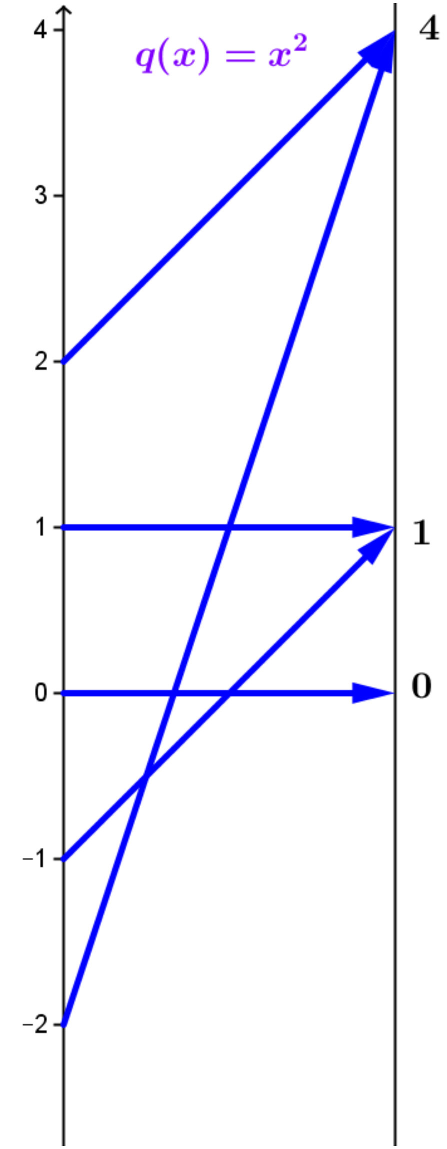
x	m(x)	f(x)=s(m(x))
2	4	5
1	2	3
0	0	1
-1	-2	-1
-2	-4	-3



Worksheet 4 Mapping Diagram:

$q(x) = x^2$

x	$q(x) = x^2$
2	4
1	1
0	0
-1	1
-2	4



Worksheet 4.a

Complete the following tables for $q(x) = x^2$ and $R(x) = s(q(x)) = x^2 + 1$

x	q(x)	R(x)=s(q(x))
2		
1		
0		
-1		
-2		

Worksheet 4.a

Complete the following tables for $q(x) = x^2$ and $R(x) = s(q(x)) = x^2 + 1$

x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5

Worksheet 4.b

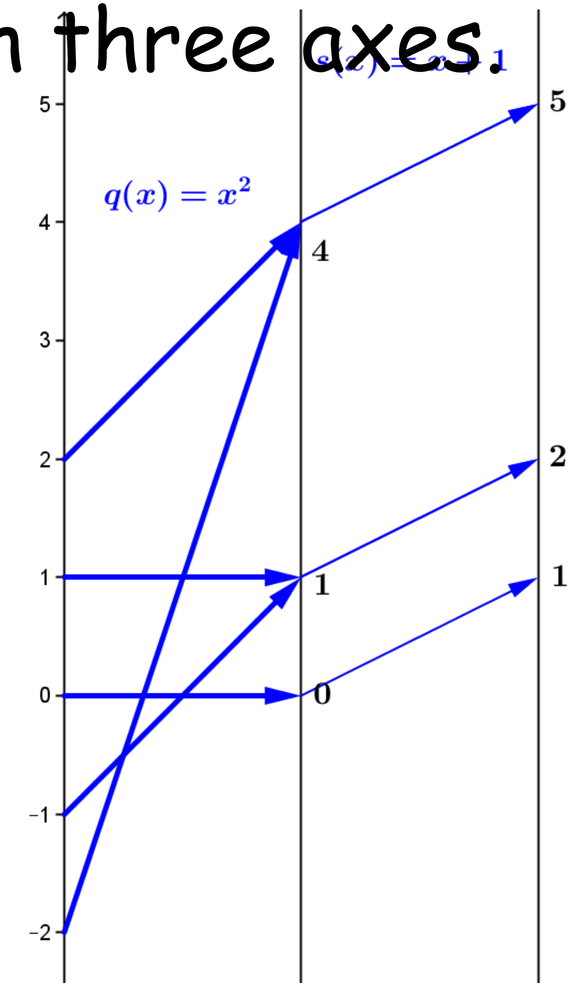
- 4.b Using the data from part a), sketch mapping diagrams for the composition $R(x) = s(q(x)) = x^2 + 1$ with three axes.

x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5

Worksheet 4.b

- 4.b Using the data from part a), sketch mapping diagrams for the composition $R(x) = s(q(x)) = x^2 + 1$ with three axes.

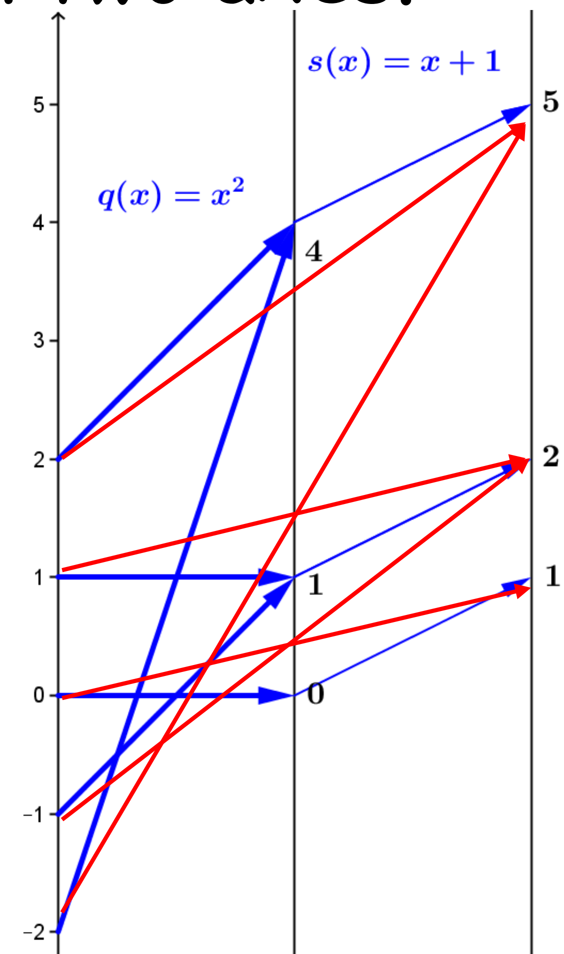
x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5



Worksheet 4.b

- 4.b Using the data from part a), sketch mapping diagrams for the composition $R(x) = s(q(x)) = x^2 + 1$ with two axes.

x	q(x)	R(x)=s(q(x))
2	4	5
1	1	2
0	0	1
-1	1	2
-2	4	5



An Old Friend: Solving A Linear Equation

- Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

Find x.

An Old Friend: Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

$$-1 = -1$$

$$2x = 4$$

An Old Friend:
Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

$$\underline{\qquad -1 = -1}$$

$$2x \quad = 4$$

$$\underline{1/2(2x) \quad = 1/2(4)}$$

$$x = 2$$

An Old Friend: Solving A Linear Equation

Worksheet 5.a Solve a linear equation:

$$2x + 1 = 5$$

$$\underline{\qquad -1 = -1}$$

$$2x = 4$$

$$\underline{1/2(2x) = 1/2(4)}$$

$$x = 2$$

Check!

$$2x+1 = 2*2 + 1 \overset{!}{=} 5$$

Linear Equations Use Linear Functions!

Linear Equations

$$2x + 1 = 5$$

$$\underline{\quad -1 = -1}$$

$$2x = 4$$

$$\underline{1/2(2x) = 1/2(4)}$$

$$x = 2$$

Check:

$$2x + 1 = 2 * 2 + 1 = 5$$

Linear Functions

$$f(x) = 2x + 1$$

Linear Equations Use Linear Functions!

Linear Equations

$$2x + 1 = 5$$

$$-1 = -1$$

$$2x = 4$$

$$1/2(2x) = 1/2(4)$$

$$x = 2$$

Check:

$$2x + 1 = 2*2 + 1 = 5$$

Linear Functions

$$f(x) = 2x + 1$$

$$m(x) = 2x;\ s(x) = x + 1$$

$$f(x) = s(m(x))$$

Worksheet 5.b Solving 2x + 1 = 5 visualized with a mapping diagram

Algebra:

2x + 1 = 5

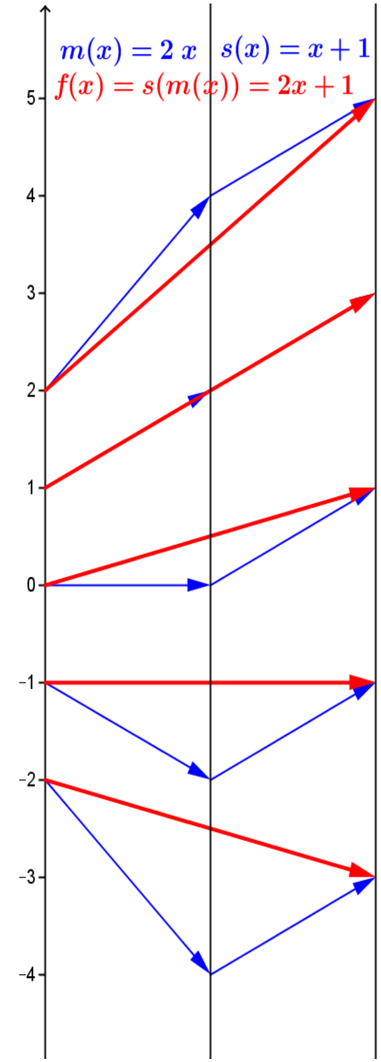
-1 = -1

2x = 4

1/2(2x) =1/2(4)

x = 2

How does the MD for the function VISUALIZE the algebra?



Worksheet 5.b Solving 2x + 1 = 5 visualized with a mapping diagram

Algebra:

$2x + 1 = 5$

$-1 = -1$

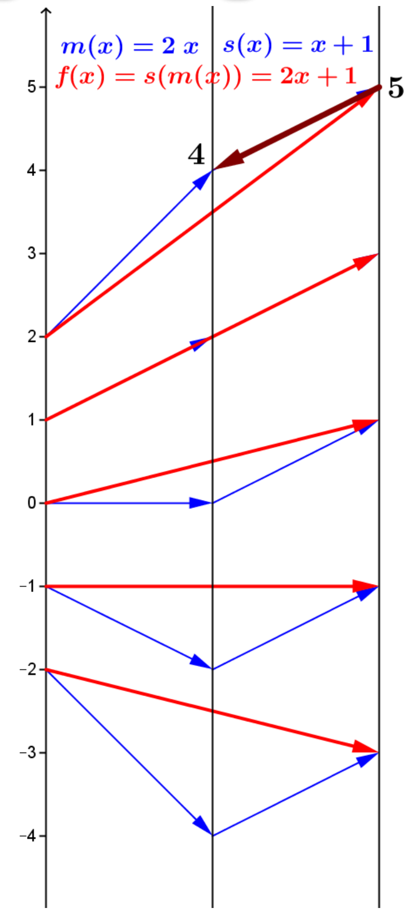
$2x = 4$

Function:

$f(x)=s(m(x)) = 5$

"Undo s"

$m(x) = 4$



Worksheet 5.b Solving 2x + 1 = 5 visualized with a mapping diagram

Algebra:

$2x + 1 = 5$

$-1 = -1$

$2x = 4$

$1/2(2x) = 1/2(4)$

$x = 2$

Function:

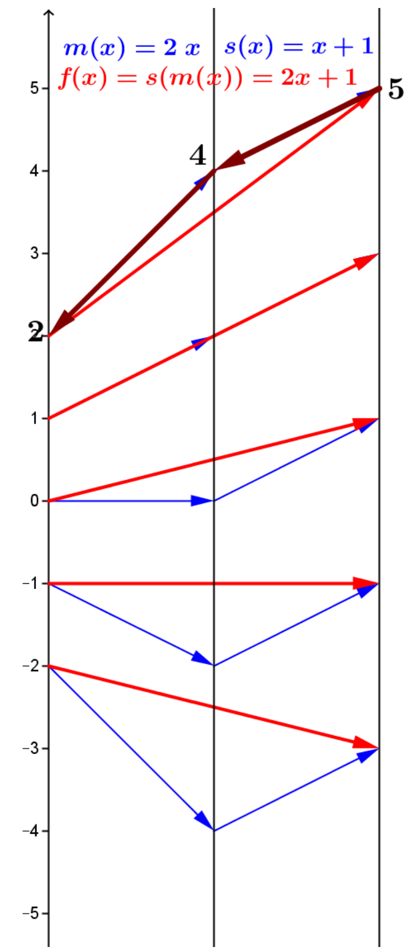
$f(x)=s(m(x)) = 5$

"Undo s"

$m(x) = 4$

"Undo m"

$x = 2$



Worksheet 5.b Solving 2x + 1 = 5 visualized with a mapping diagram

Algebra:

$2x + 1 = 5$

$-1 = -1$

$2x = 4$

$1/2(2x) = 1/2(4)$

$x = 2$

Function:

$f(x)=s(m(x)) = 5$

"Undo s"

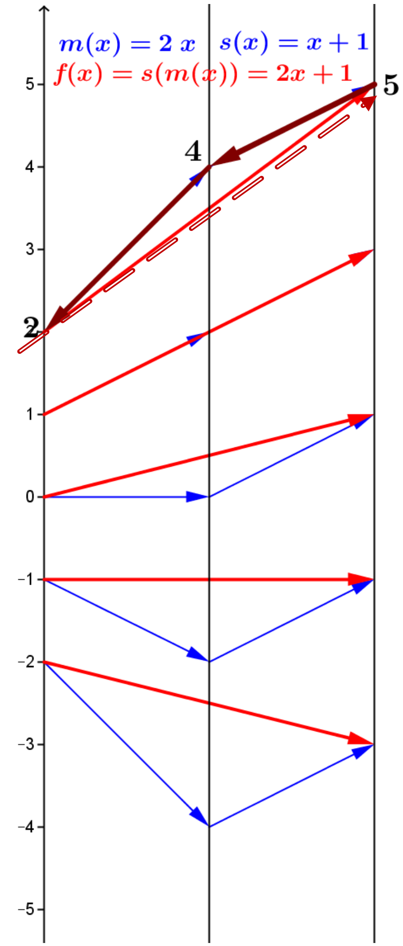
$m(x) = 4$

"Undo m"

$x = 2$

CHECK! ☺

$f(2) \overset{!}{=} 5$



Worksheet 5.b Solving 2x + 1 = 5 visualized on GeoGebra

Algebra:

2x + 1 = 5

<u>-1 = -1</u>

2x = 4

<u>1/2(2x) =1/2(4)</u>

x = 2

Function:

f(x)=s(m(x)) = 5

"Undo s"

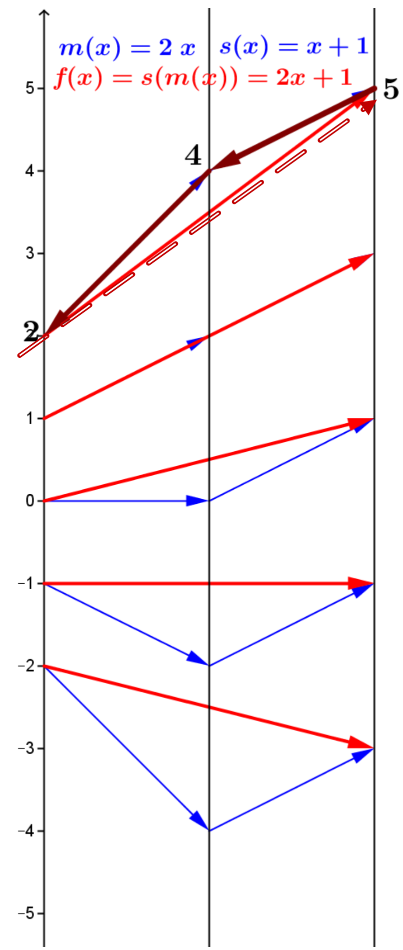
m(x) = 4

"Undo m"

x = 2

CHECK! ☺

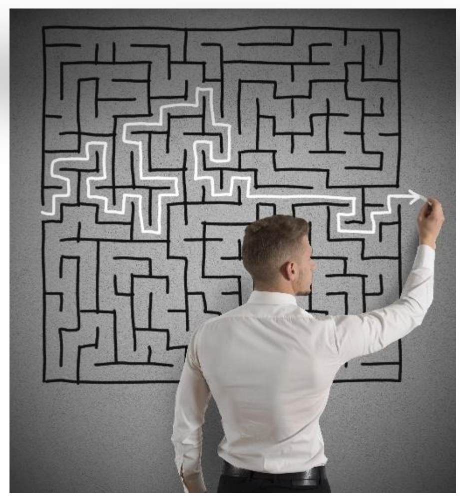
$$f(2) \overset{!}{=} 5$$



Challenge: Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

The Great Mind Challenge

Initiate Collaborate Innovate



Worksheet 6.a Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Understand the problem

- $2(x-3)^2 + 1$ is a function of x.
  - $P(x) = 2(x-3)^2 + 1$
- Find any and all x where $P(x) = 9$.
- $2(x-3)^2 + 1$ is a composition of functions
  - $P(x) = s(m(q(z(x))))$ where
  - $z(x) =$
  - $q(x) =$
  - $m(x) =$
  - $s(x) =$

Worksheet 6.a Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Understand the problem

- $2(x-3)^2 + 1$ is a function of x.
  - $P(x) = 2(x-3)^2 + 1$
- Find any and all x where $P(x) = 9$.
- $2(x-3)^2 + 1$ is a composition of functions
  - $P(x) = s(m(q(z(x))))$ where
  - $z(x) = x-3$;
  - $q(x) = x^2$;
  - $m(x) = 2x$;
  - $s(x) = x+1$.

Worksheet 6.a Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram.

Make a plan

- Find any and all x where $P(x) = 9$.
- Construct mapping diagram for P as a composition of function :

$$P(x) = s(m(q(z(x))))$$

- Undo $P(x) = 9$ by undoing each step of P
  - Undo $s(x) = x+1$
  - Undo $m(x) = 2x$
  - Undo $q(x) = x^2$
  - Undo $z(x) = x-3$
- Check results to see that $P(x) = 9$

Worksheet 6.b Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram.

Execute the **plan**

- Construct mapping diagram for P as a composition of function :

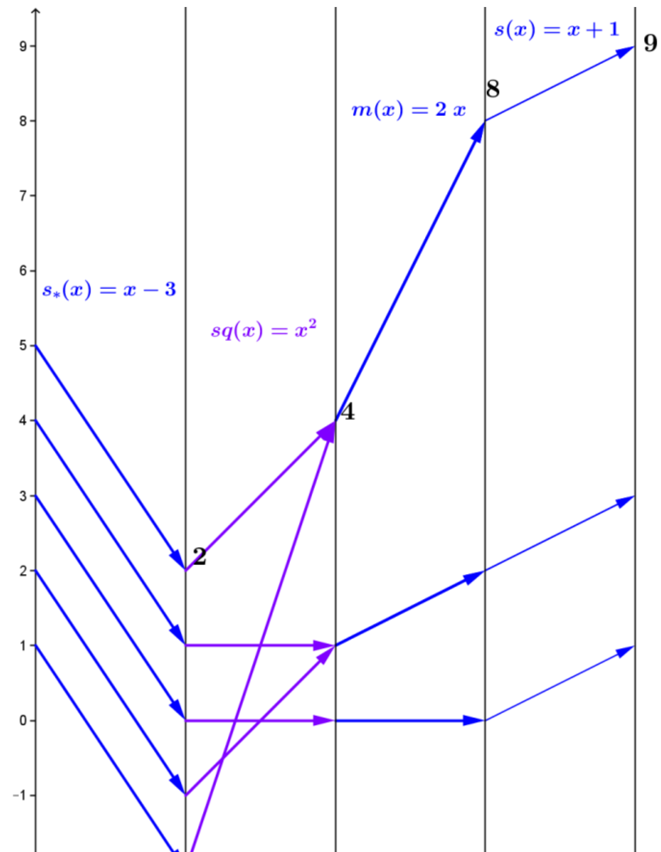
$$P(x) = s(m(q(z(x))))$$

Worksheet 6.b Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram.

Execute the **plan**

- Construct mapping diagram for P as a composition of function :

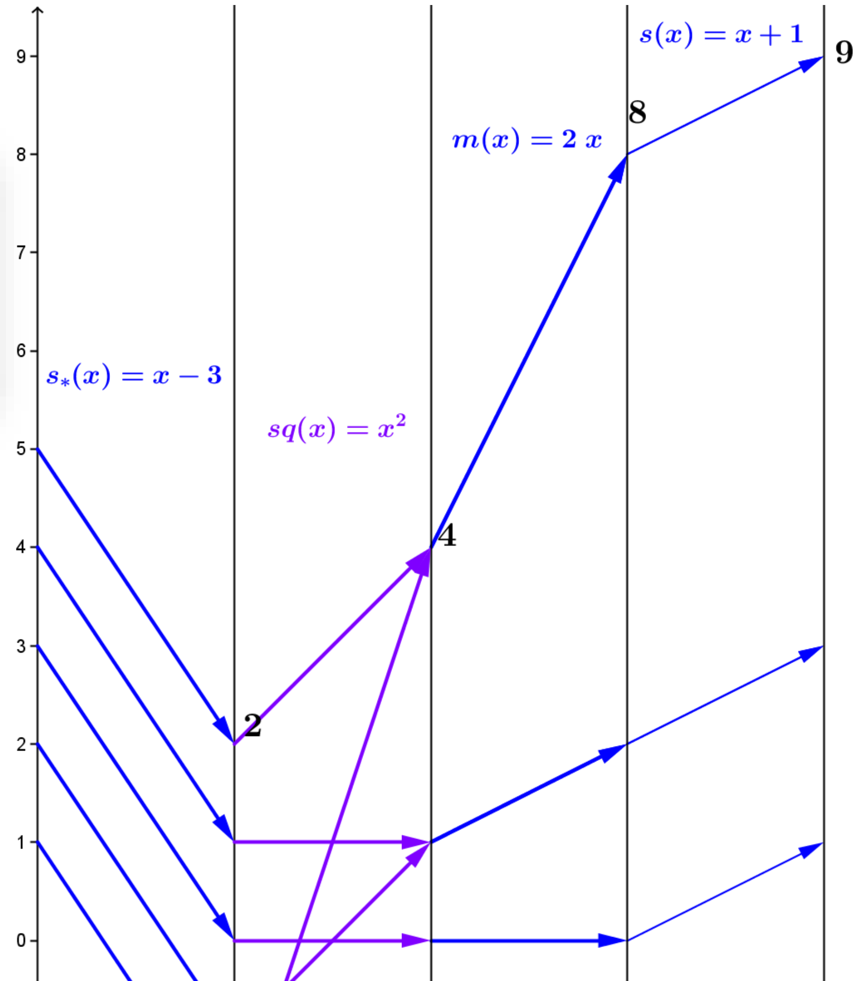
$$P(x) = s(m(q(z(x))))$$



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Execute the plan

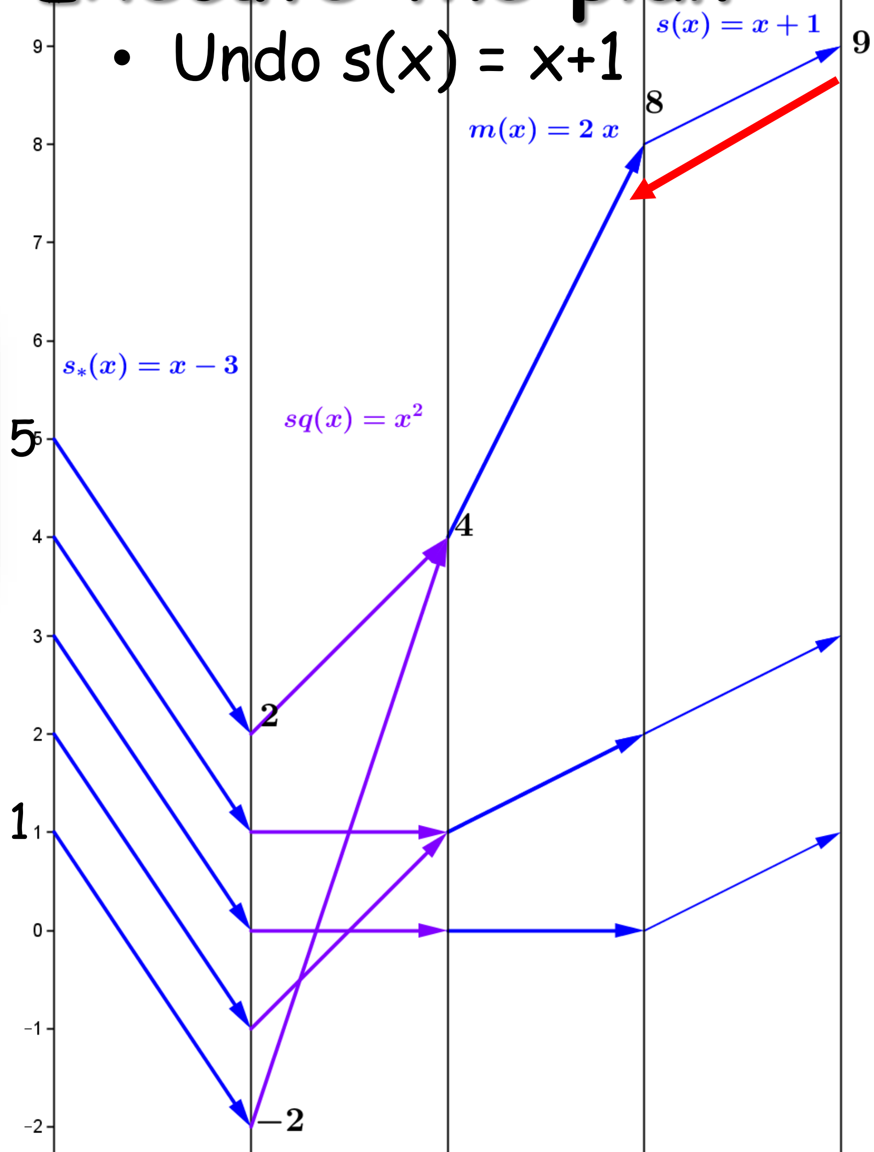
- Find any and all x where P(x) = 9.



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Execute the plan

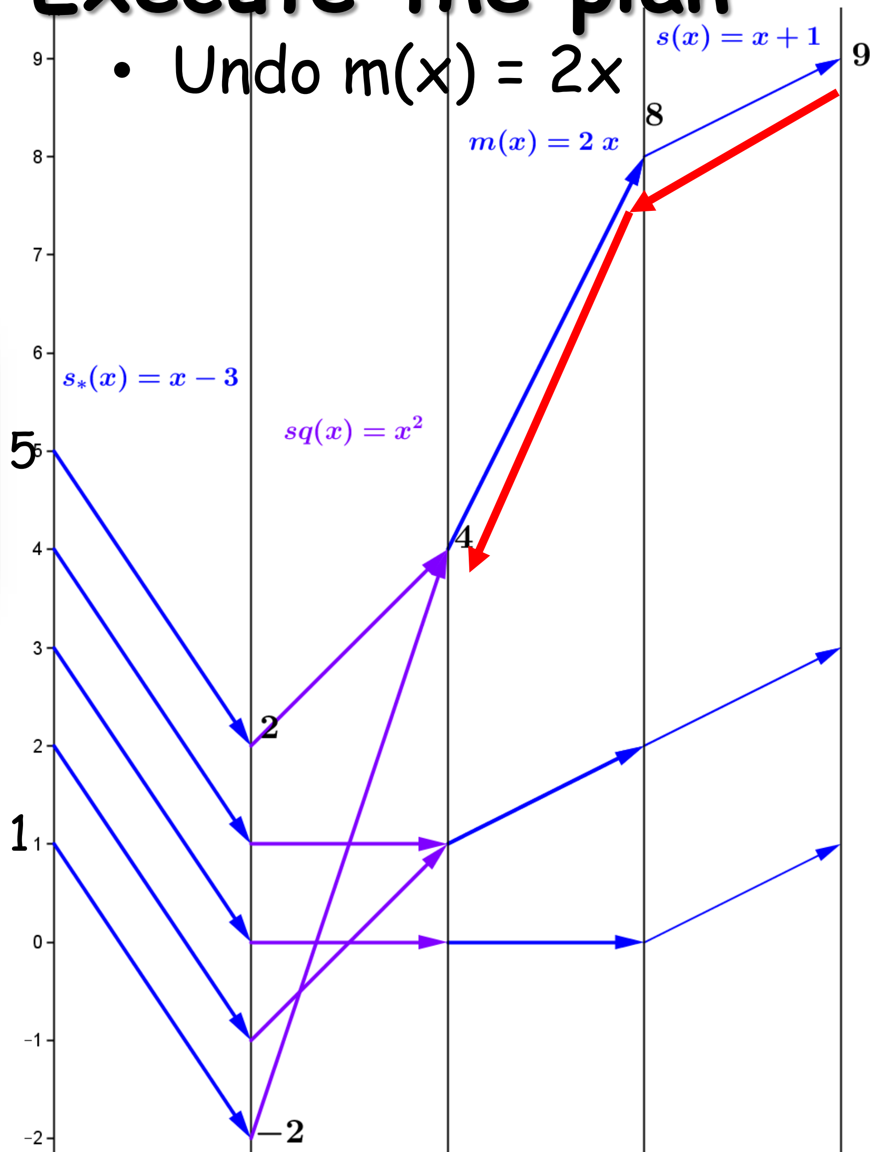
- Undo $s(x) = x+1$



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Execute the plan

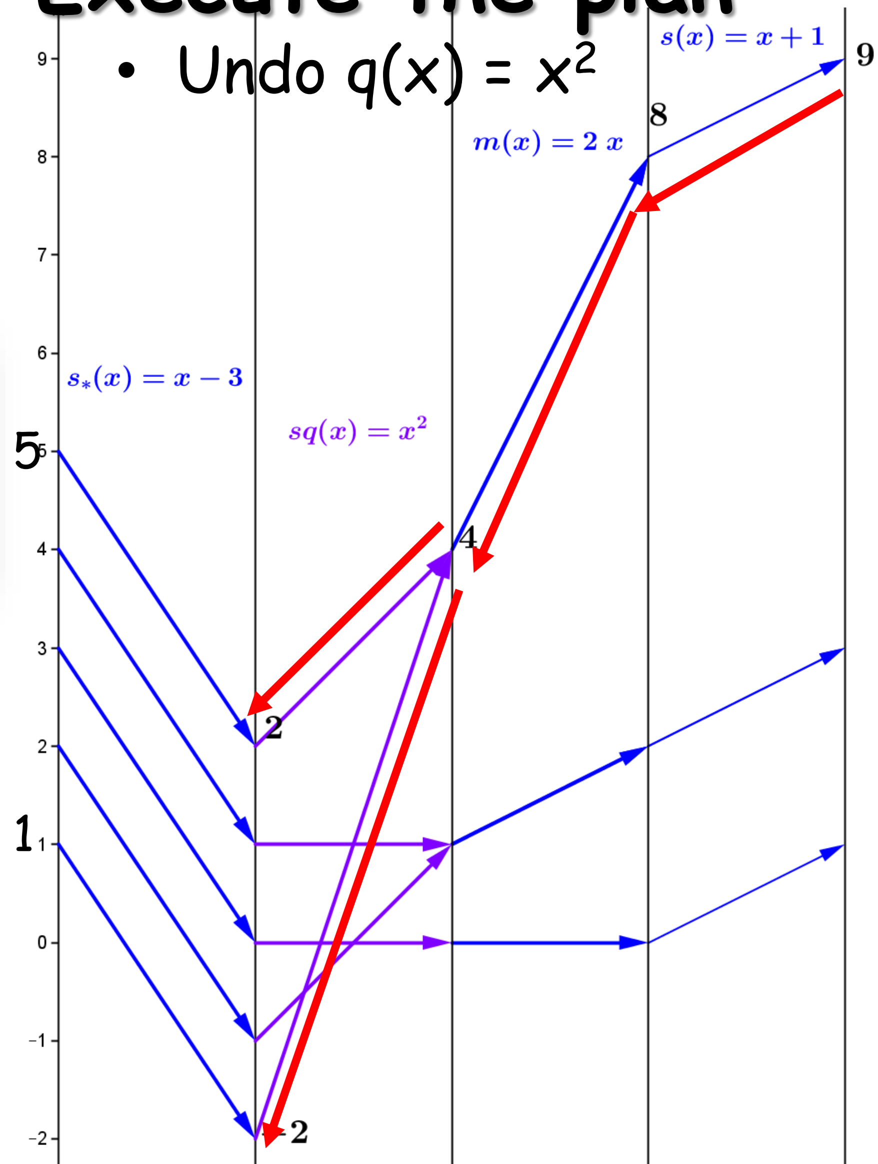
- Undo $m(x) = 2x$



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Execute the plan

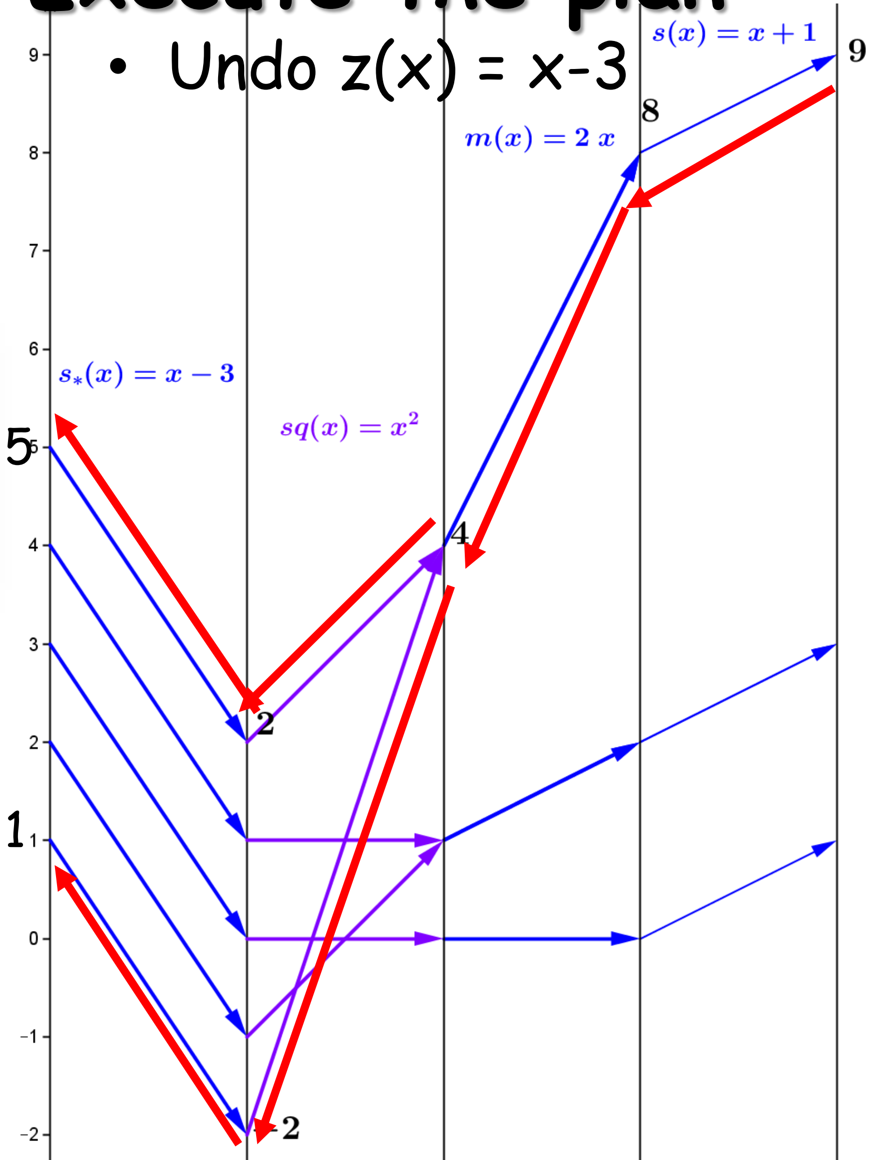
- Undo $q(x) = x^2$



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

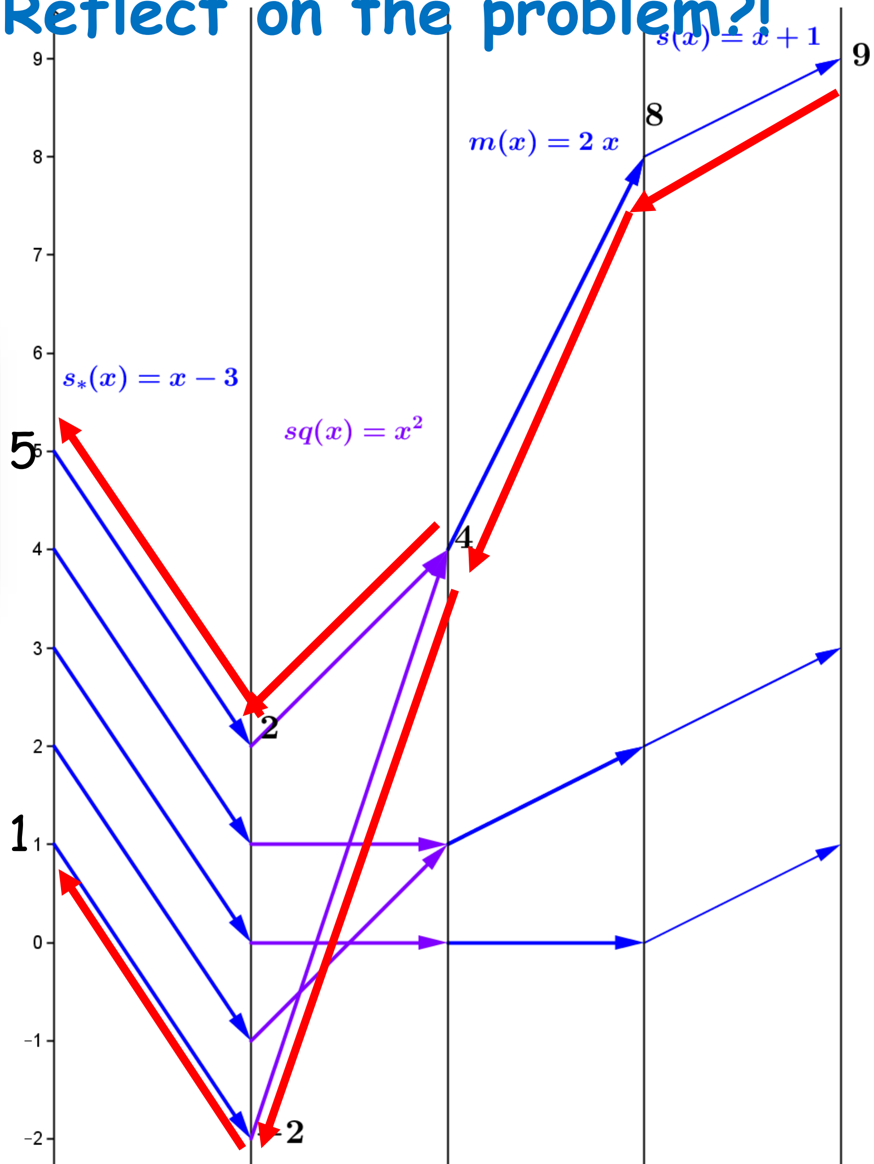
Execute the plan

- Undo $z(x) = x-3$



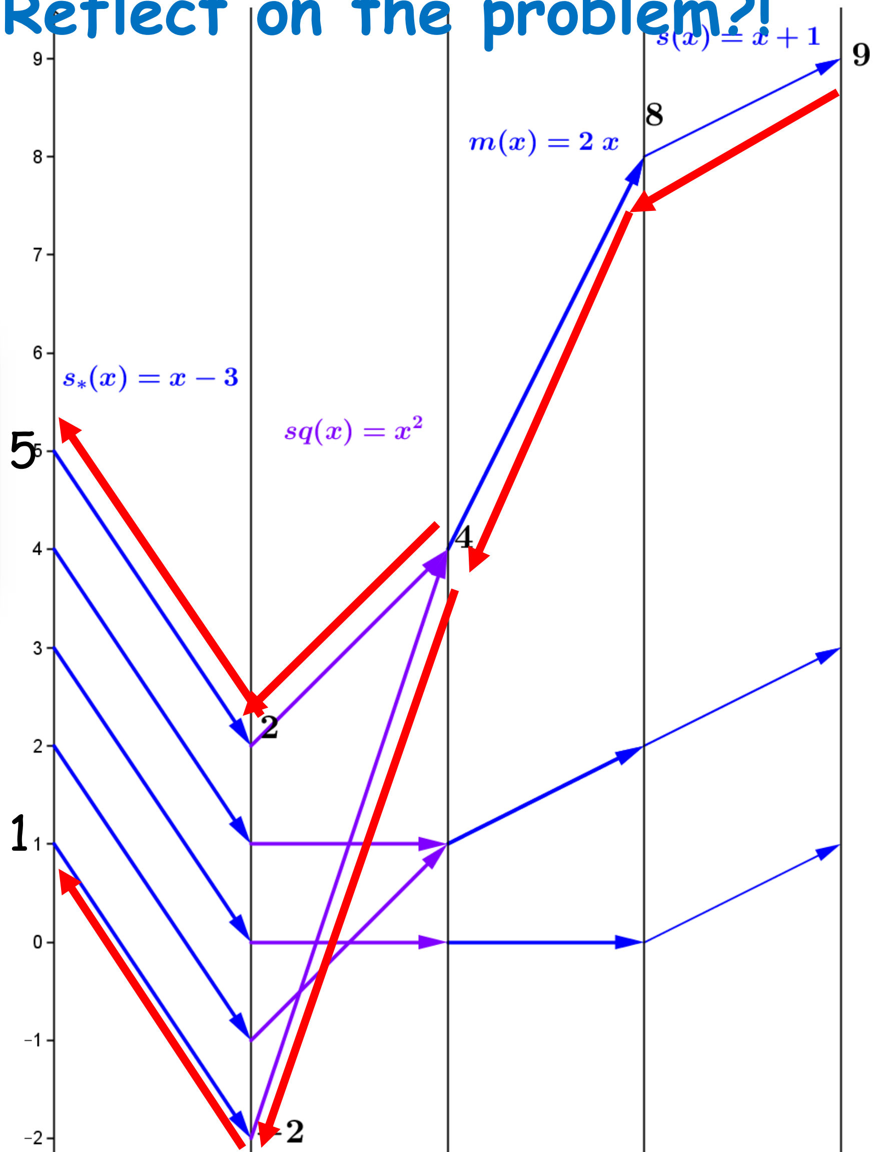
Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Reflect on the problem?!



Worksheet 6.c Solve $2(x-3)^2 + 1 = 9$ with a mapping diagram

Reflect on the problem?!



Technology Examples

- Excel examples
- Geogebra examples

Simple Examples are important!

- $f(x) = x + C$ Added value: C
- $f(x) = mx$ Scalar Multiple: m

  Interpretations of m:
  - slope
  - rate
  - Magnification factor
  - $m > 0$: Increasing function
  - $m < 0$: Decreasing function
  - $m = 0$: Constant function

Simple Examples are important!

$f(x) = mx + b$ with a mapping diagram --

Five examples:

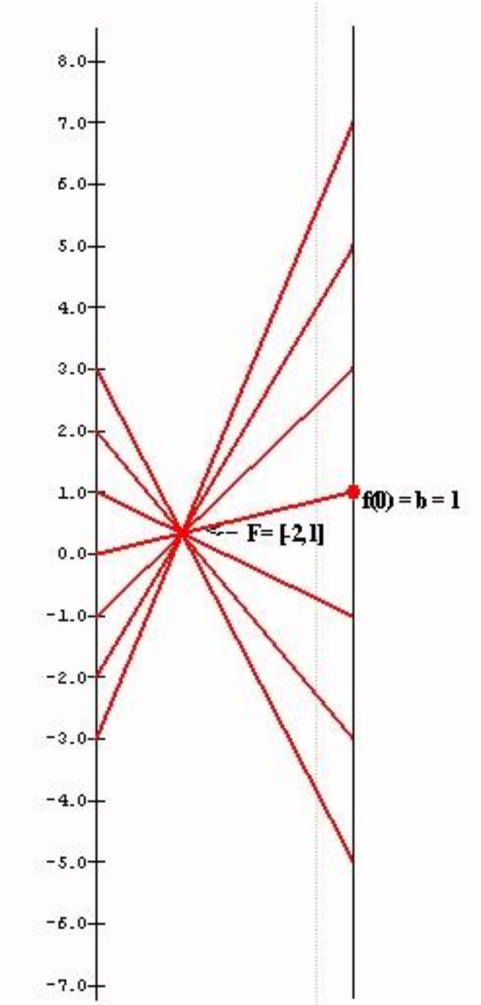
Back to Worksheet Problem #7

- Example 1: $m = -2$; $b = 1$: $f(x) = -2x + 1$
- Example 2: $m = 2$; $b = 1$: $f(x) = 2x + 1$
- Example 3: $m = \frac{1}{2}$; $b = 1$: $f(x) = \frac{1}{2} x + 1$
- Example 4: $m = 0$; $b = 1$: $f(x) = 0 x + 1$
- Example 5: $m = 1$; $b = 1$: $f(x) = x + 1$

Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

Example 1: m = -2; b = 1

$f(x) = -2x + 1$



- Each arrow passes through a single point, which is labeled **F = [- 2,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $-2x + 1$,

which corresponds to the linear function's value for the point/number, *x*.

Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

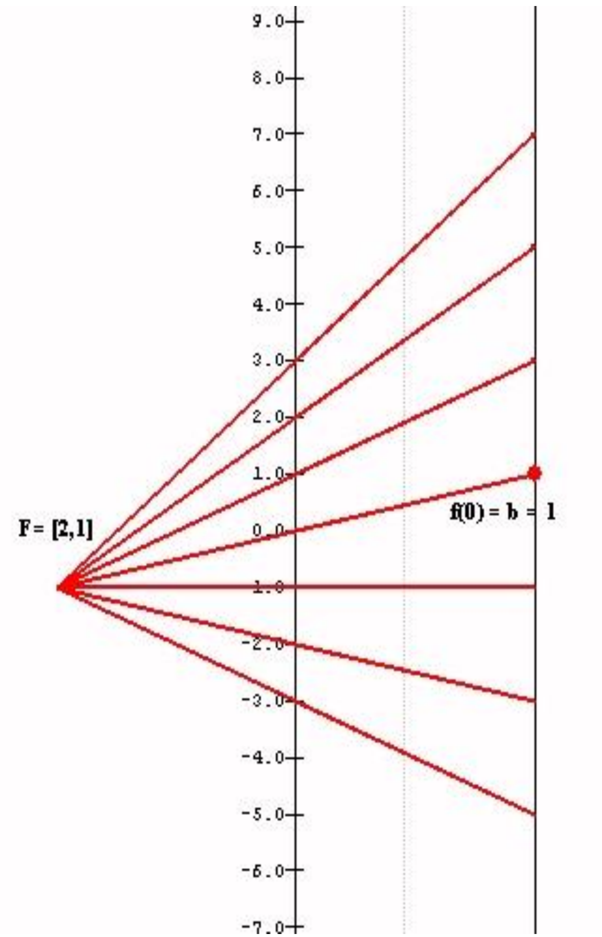
Example 2: m = 2; b = 1
f(x) = 2x + 1

Each arrow passes through a single point, which is labeled

F = [2,1].

- The point **F** completely determines the function *f*.
  - **given a point** / number, *x*, on the source line,
  - there is a **unique arrow passing through F**
  - **meeting** the target line at **a unique point** / number, $2x + 1$,

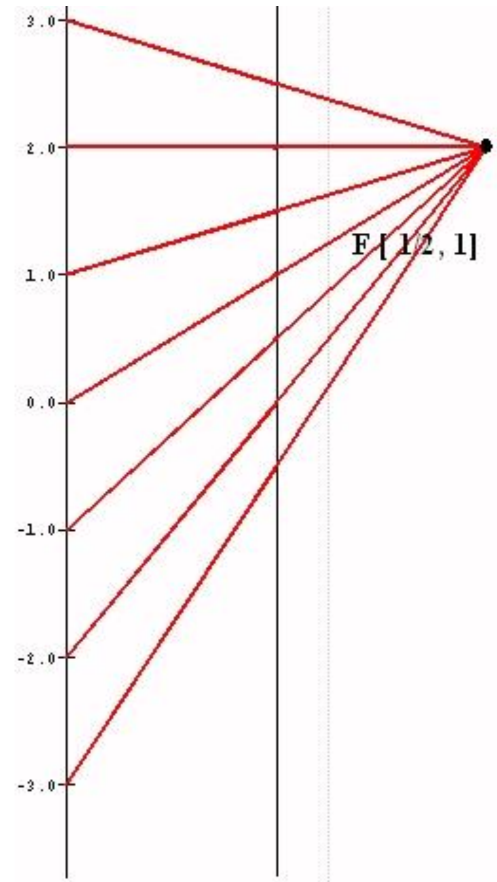
which corresponds to the linear function's value for the point/number, *x*.



Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

- **Example 3: $m = 1/2$; $b = 1$**
  $f(x) = \frac{1}{2}x + 1$
- Each arrow passes through a single point, which is labeled **F = [1/2,1]**.
  - The point **F** completely determines the function f.
    - **given a point** / number, x, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $\frac{1}{2}x + 1$,

    which corresponds to the linear function's value for the point/number, x.

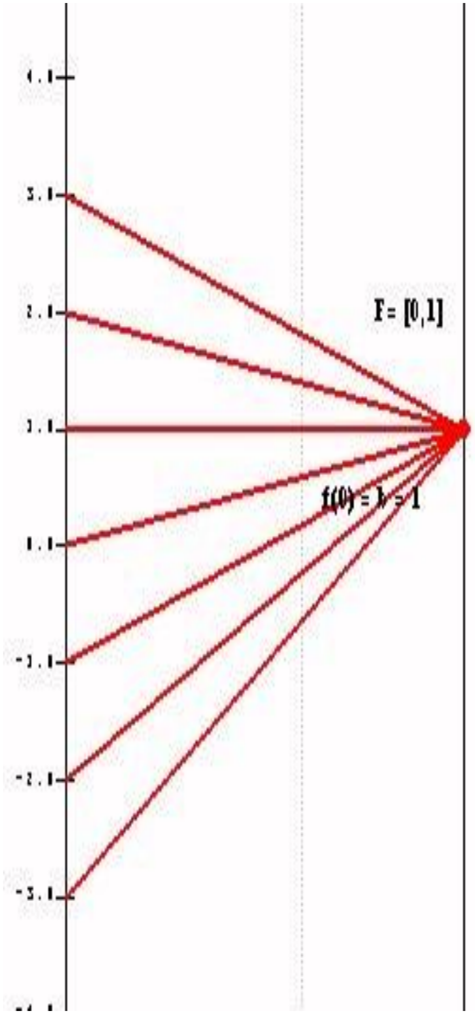


Visualizing $f(x) = \boldsymbol{mx} + \boldsymbol{b}$ with a mapping diagram -- Five examples:

- **Example 4: $m = 0$; $b = 1$**
  $f(x) = 0\ x + 1$

- Each arrow passes through a single point, which is labeled **F = [0,1]**.
  - The point **F** completely determines the function f.
    - **given a point** / number, x, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $f(x)=1$,

    which corresponds to the linear function's value for the point/number, x.



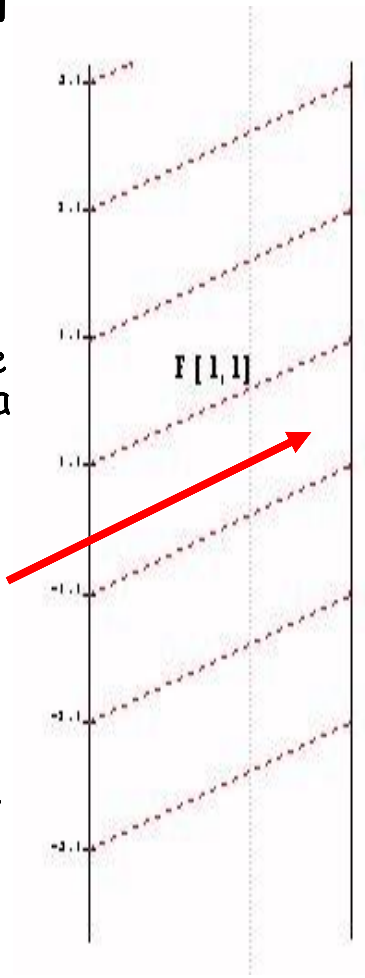
Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples

Example 5: $\boldsymbol{m} = 1$; $\boldsymbol{b} = 1$

$f(x) = x + \boldsymbol{1}$

- Unlike the previous examples, in this case it is not a single point that determines the mapping diagram, but the single arrow from 0 to 1, which we designate as F[1,1]
- It can also be shown that this single arrow completely determines the function.Thus, given a point / number, x, on the source line, there is a unique arrow passing through ***x parallel to*** F[1,1] meeting the target line a unique point / number, $x + 1$, which corresponds to the linear function's value for the point/number, x.
  - The single arrow completely determines the function f.
    - **given a point** / number, x, on the source line,
    - there is a **unique arrow** through ***x parallel to*** F[1,1]
    - **meeting** the target line at **a unique point** / number, $x + 1$,

which corresponds to the linear function's value for the point/number, x.



Simple Examples are important!

- $f(x) = x + C$ Added value: C
- $f(x) = mx$ Scalar Multiple: m

  Interpretations of m:
  - slope
  - rate
  - Magnification factor
  - $m > 0$: Increasing function
  - $m < 0$: Decreasing function
  - $m = 0$: Constant function

Function-Equation Questions

with linear focus points (Problem 8.a)

- Use a focus point in the mapping diagram to solve a linear equation:

$$2x+1 = 5$$

Function-Equation Questions

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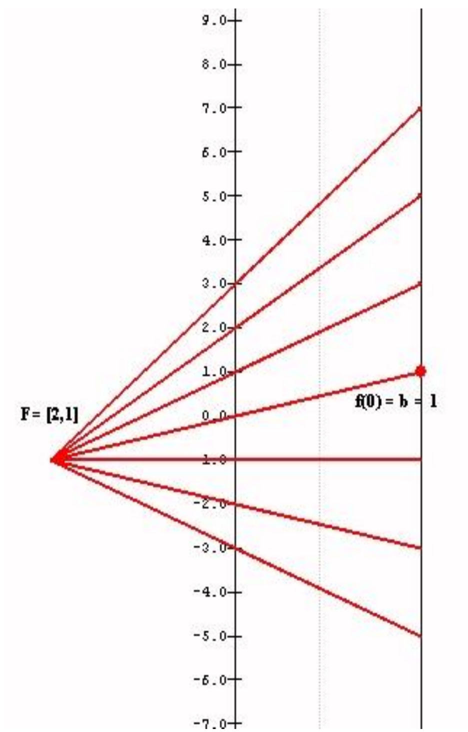

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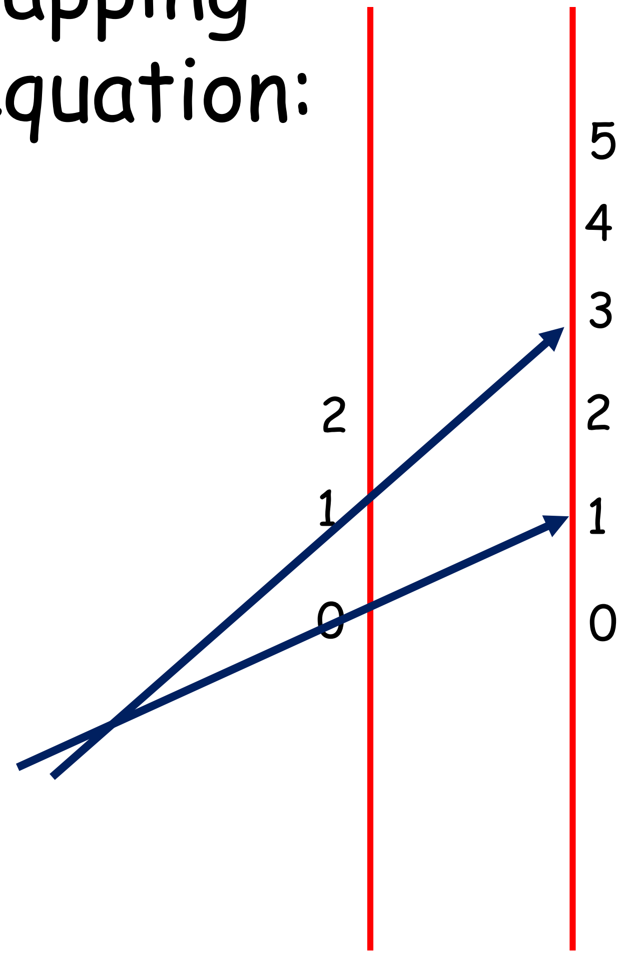

Function-Equation Questions

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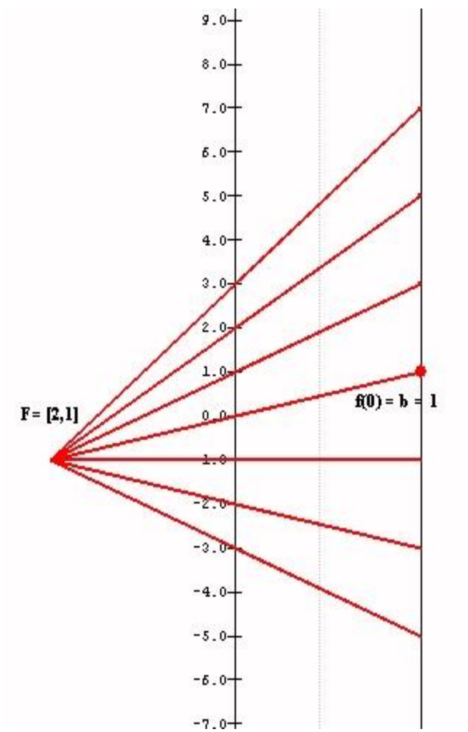
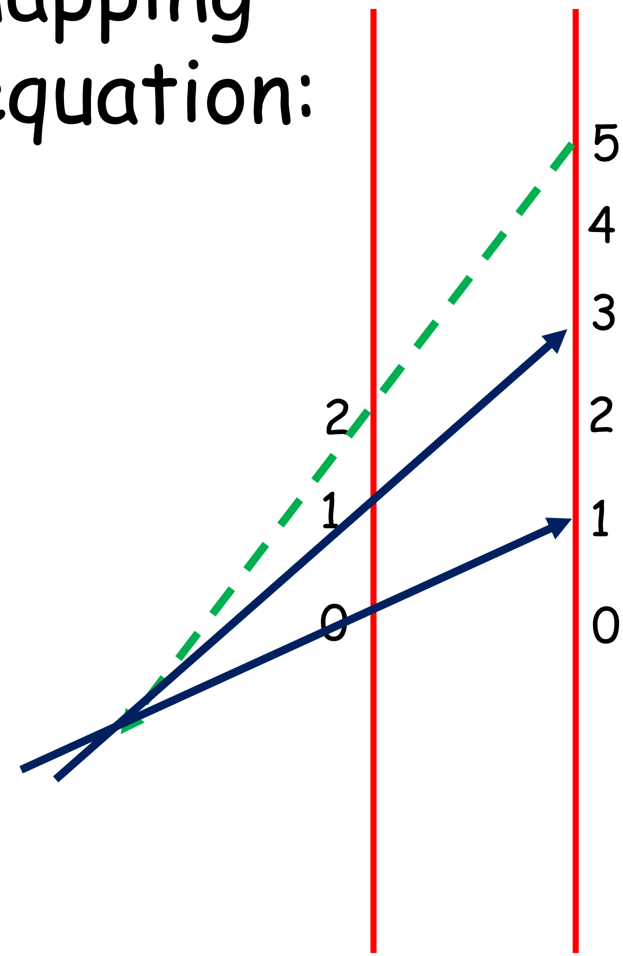




Function-Equation Questions

with linear focus points (Problem 8)

Suppose f is a linear function with f (1) = 3 and f (3) = -1.

Without algebra

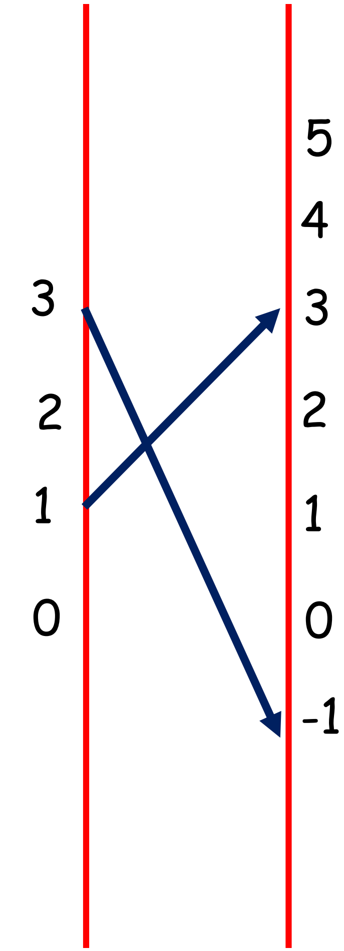
- 8.b Use a focus point to find f (0).
- 8.c Use a focus point to find x where f (x) = 0.

Function-Equation Questions
with linear focus points (Problem 8)

Suppose f is a linear function with $f(1) = 3$ and $f(3) = -1$.

Without algebra

- 8.b Use a focus point to find $f(0)$.
- 8.c Use a focus point to find x where $f(x) = 0$.



Function-Equation Questions

with linear focus points (Problem 8)

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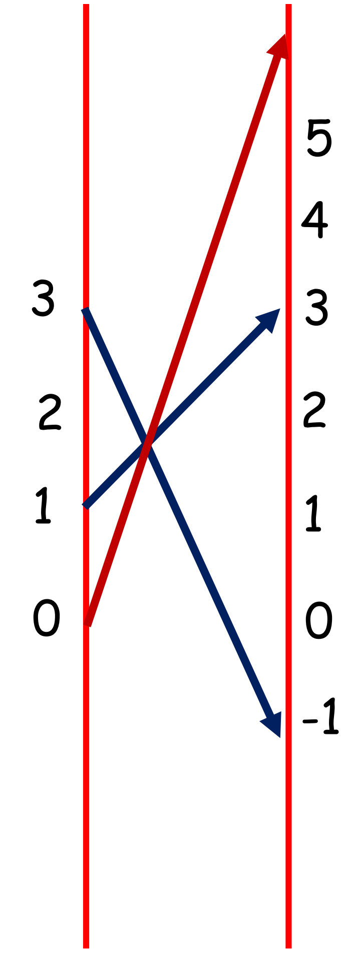
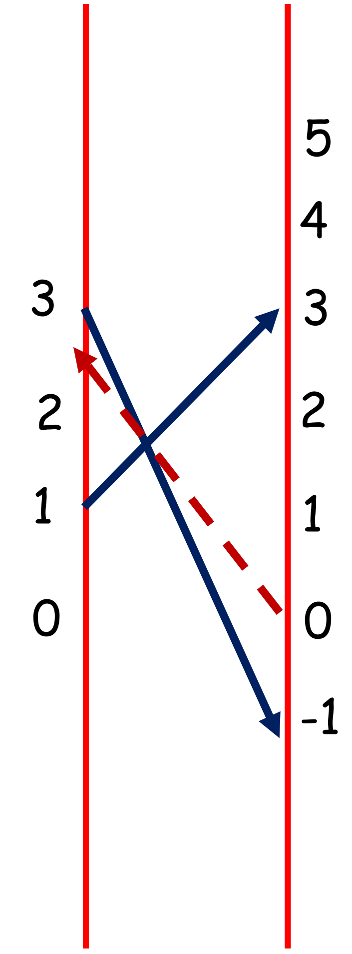


Function-Equation Questions
with linear focus points (Problem 8)

Suppose f is a linear function with f (1) = 3 and f (3) = -1.

Without algebra

- 8.c Use a focus point to find x where f (x) = 0.



- Evaluations – Survey Monkey.
- Please fill out the paper version found in your bags or online. The url is https://www.surveymonkey.com/r/CMC-NorthSpeakerEvaluations
- The QR code is

Thanks

The End!

☺

Questions?

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References

- Solving Linear Equations Visualized with Mapping Diagrams (YouTube) by M. Flashman

- **Function Diagrams. by Henri Picciotto Excellent Resources!**
  - **Henri Picciotto's Math Education Page**
  -

- Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s. A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET FOR publication) **http://users.humboldt.edu/flashman/MD/section-1.1VF.html**

- **Mapping Diagrams and Graphs…** Visualizing linear functions using mapping diagrams and graphs. **tube.geogebra.org** Martin Flashman

Thanks

The End!   REALLY!

☺

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