Making Sense of Integration Visually:
Mapping Diagrams for Calculus   BCME H20 Worksheet   Martin Flashman ©2018

1.
   a.   Complete the following tables for $m(x) = 2x$ and $s(x) = x + 1$

x	$m(x) = 2x$	$s(x) = x + 1$
2		
1		
0		
-1		
-2		

   b.   Using the data from part a), on separate diagrams sketch mapping diagrams for $m(x) = 2x$ and $s(x) = x+1$
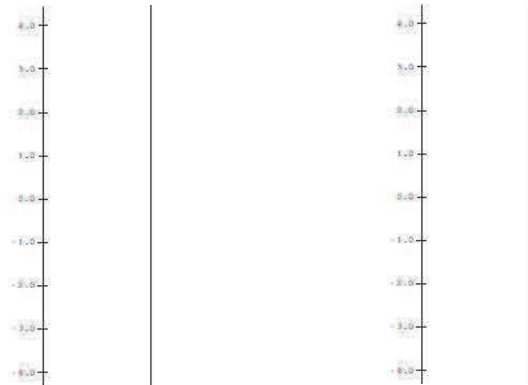
2.   Let $q(x) = x^2$.

   a. Complete the following table for $q(x) = x^2$.

x	$q(x) = x^2$
2	
1	
0	
-1	
-2	

   b. Using the data from part a), sketch a mapping diagram for $q(x) = x^2$.

3. Let $f(x) = mx + b$ sketch mapping diagrams for the following:
Use the same scale for the second axis.
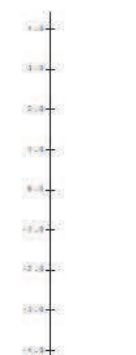
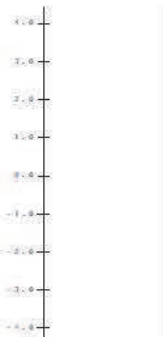 a. $m = -2; b = 1: f(x) = -2x + 1;$

  d. $m = 0; b = 1: f(x) = 0 \, x + 1$
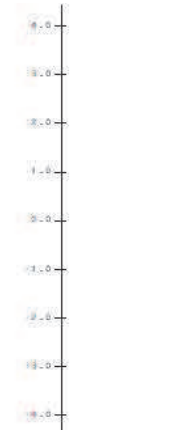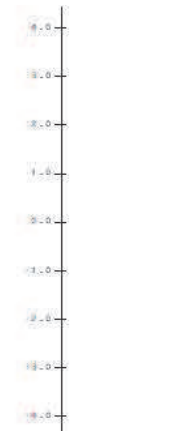
  b. $m = 2 \; b = 1: f(x) = 2x + 1$

  e. $m = 1; b = 1: f(x) = x + 1$

  c. $m = \frac{1}{2}; b = 1: f(x) = \frac{1}{2} x + 1$

4. Let $f(x) = x^2 - 1$. Use a mapping diagram to visualize estimating the values of $f(1.1)$ and $f(0.9)$ with the differential. [Use $d\,x = \pm 0.1$, near the value for $x = 1$ where $f(1) = 0$, and $dy = f'(1) * dx$.]

5. Complete the following table to estimate of the solution $f(2)$ of the following initial value problem by Euler's method with $n = 4$ ($\Delta x = \frac{1}{2}$). Use a mapping diagram to visualize the result.

$$\frac{dy}{dx} = f'(x) = 2x - 1 \text{ with } f(0) = 1.$$

x	$f(x)$	$\dfrac{dy}{dx} = f'(x) = 2x - 1$	$dy = f'(x)dx = (2x-1)dx$
0	1		
1/2			
1			
3/2			
2			

6.

a. Complete the following table to estimate $\int_0^2 2t-1\,dt$ by an Euler's sum with $n = 4$ ($\Delta t = \frac{1}{2}$).

Use a mapping diagram to visualize the result.

x	$\int_0^x 2t-1\,dt$	$P(t)=2t-1$	$P(t)\Delta t=(2t-1)\Delta t$
0			
1/2			
1			
3/2			
2			

b. Suppose f is a solution to $\dfrac{dy}{dt}=f'(t)=2t-1$.

Use the estimate for $\int_0^2 2t-1\,dt$ to estimate $f(2) - f(0)$.

c. Discuss the connection of this estimate to the mapping diagram and problem 5.

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7. Complete the following tables for $P(x)$ to estimate $\int_1^3 f'(t)\,dt$ using Euler's Method and visualize the estimation with a mapping diagram.

x	$\int_1^x f'(t)\,dt$	$f'(t)$	$f'(t)\Delta t$
1		4	
1.5		2	
2		3	
2.5		5	
3			

8. Visualize the additive property of the definite integral, $\int_a^c P(x)\,dx+\int_c^b P(x)\,dx = \int_a^c P(x)\,dx$, with a mapping diagram with $P(x) = 2x$, $a = 1$, $b = 3$, $c = 2$.

Use your knowledge of the FT of C to find the actual integrals.

$\int_1^2 2x\,dx =$ _____

$\int_2^3 2x\,dx =$ _____

$\int_1^3 2x\,dx =$ _____

Adjust the scale for the target as needed to make the diagram fit on the given axes.

9. Visualize the scalar multiplication property of the definite integral,
$\int_a^b \alpha\, P(x)\, dx = \alpha \int_a^b P(x)\, dx$, with a mapping diagram with $P(x) = 2x$, $a = 0$, $b = 2$, $\alpha = 3$.

Explain your figure's connection to the equation.

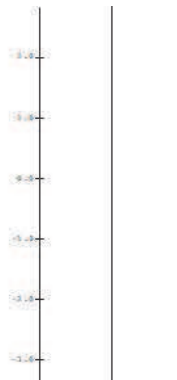Use your knowledge of the FT of C to find the two integrals.

$\int_0^2 2x\, dx = $ _____

$\int_0^2 3 \cdot 2x\, dx = $ _____

Adjust the scale for the three axes as needed to make the diagram fit on the given axes.

Place the integral value for $\int_0^2 2x\, dx$ on the middle axis.

Place the integral value for $\int_0^2 3 \cdot 2x\, dx$ on the right most axis.

Apply the linear function $m(x) = 3x$ to the value on the middle axis .

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10. Visualize the mean value property of the definite integral, $\int_a^b P(x)\, dx = P(c) \cdot (b-a)$, with a

mapping diagram with $P(x) = 2x$, $a = 0$, $b = 2$. Explain your figure's connection to the equation.

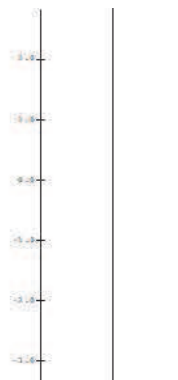Use your knowledge of the FT of C to find the integral.

$\int_0^2 2x\, dx = $ _____

Visualize the function P between the first and middle axes.
Find Max and min - the max and min values for P on the middle axis.
Adjust the scale for the three axes as needed to make the diagram fit on the given axes.

Place the integral value on the third axis.
Apply the linear function $m(x) = (b-a)\, x = 2\, x$ to the values on the middle axis .Indicate where c lies on the first axis.

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