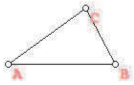
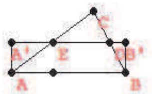
1. Quadrature Problem: Given a region in the plane, find a root so that the square of this root has the same area.
   a. Quadrature of a Triangle:Given ΔABC ,

Find a square ☐ DEFG with root = DE and area of ☐ DEFG = area of ΔABC

i. Construct a rectangle equal in area to that of ΔABC



ii. Construct a square equal in area to the rectangle.



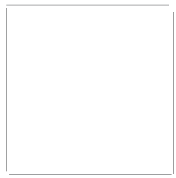
b. Quadrature for Polygons:

Problem: Find the root of a square that has the same area as a given polygon.
Suggest the outline for a procedure to accomplish the solution of the problem..
Hint: Use triangles and the Pythagorean Theorem.

2. Example for completing the square problem:
[al'Khowarizmi ≈820 AD and al'Khayyam≈1100 AD.]
Find the root of the square which when added to a rectangle with one side of the same length as the root gives a rectangle of area *c*.

x   *b*

has area *c*:



$x^2+bx=c$, has root $x=\sqrt{(b/2)^2+c}-b/2$

3. Descartes Arithmetic for Segments:
   a. Multiplication using a unit segment and proportional sides of similar triangles.

   b. Square roots using a unit segment and right triangles in a semicircle.

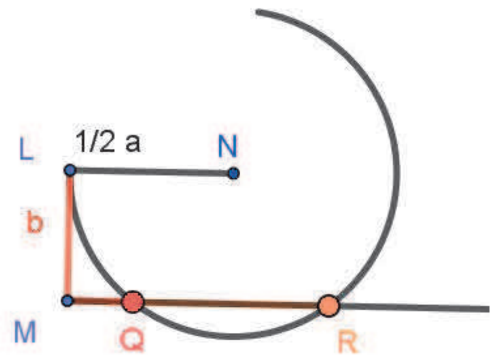
4. Descartes Arithmetic for Line Segments;
   a. Multiplication

   b. Square Roots

5. Descartes solves a quadratic equation for the arithmetic of segments.

$$z^2 = az - b^2$$

NL = 1/2 a, LM = b, NL⊥LM. MQR | | LN
Circle with center N , through L, meeting MQR at Q and R.
Show that MQ and MR are solutions for z in the equation. [Hint:Use the Pythagorean Theorem]



6. Suppose $f(x) = x^2 - 4x + 2$
   a. Draw a sketch of the graph $g(x) = f(x)$ - 2 by finding the roots of g.

   b. Find the axis of symmetry for g and f.

   c. Express f in the vertex form (“completing the square”).

   d. Solve the equation: $f(x) = x^2 - 4x + 2 = 0.$

7.

a. Complete the following tables for $m(x) = 2x$ and $s(x) = x + 1$

x	$m(x) = 2x$	$s(x) = x + 1$
2		
1		
0		
-1		
-2		

b. Using the data from part a), on separate diagrams sketch mapping diagrams for $m(x) = 2x$ and $s(x) = x+1$

8. Let $q(x) = x^2$.

a. Complete the following table for $q(x) = x^2$.

x	$q(x) = x^2$
2	
1	
0	
-1	
-2	

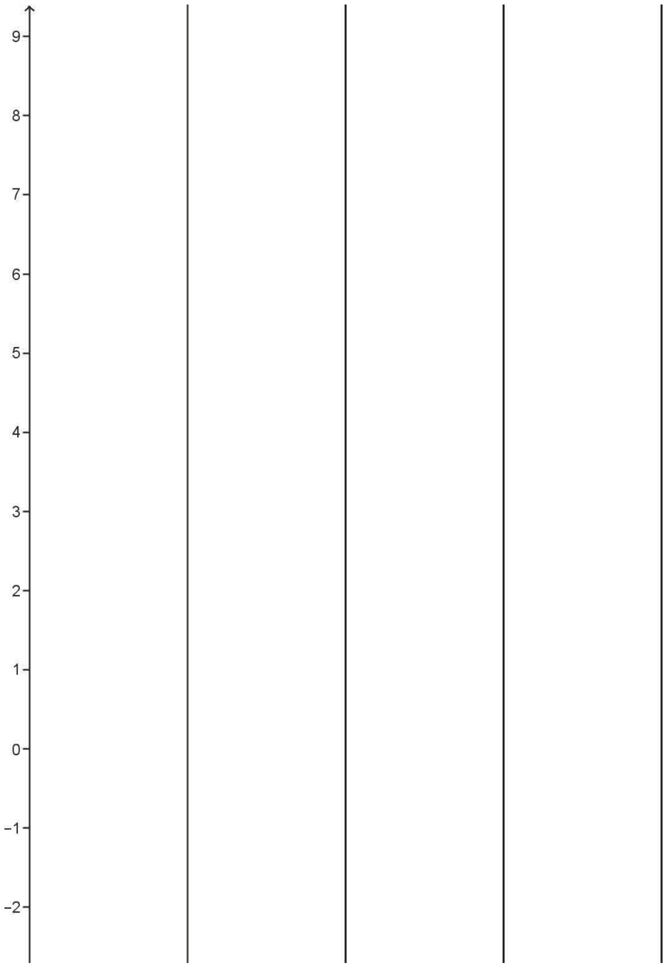
b. Using the data from part a), sketch a mapping diagram for $q(x) = x^2$.

9. Solving $\mathbf{2(x\text{-}3)^2 + 1 = 9}$ with a mapping diagram.
   a. Express $\boldsymbol{f(x)} = \mathbf{2(x\text{-}3)^2 + 1}$ as composition of core linear and quadratic functions.

      $f(x) = h(m(q(z(x))))$ where

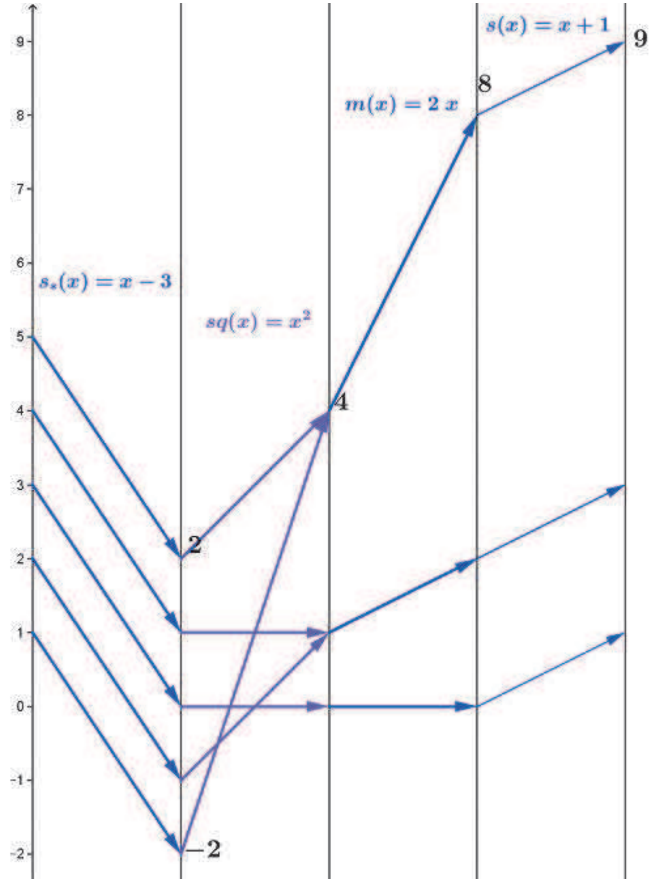
      $h(x) =$ ____________
      $m(x) =$ ____________
      $q(x) =$ ____________
      $z(x) =$ ____________

   b. Sketch a mapping diagram for *f* **as a composition**.



How Many Ways Can You Solve a Quadratic Equation Visually?
From the Greeks to 21st Century Technology   Worksheet   Martin Flashman ©2018

   c. On the mapping diagram below indicate by circling numbers and arrows how the diagram visualizes the solution of $\mathbf{2(x\text{-}3)^2 + 1 = 9}$. **Check the solutions.**



Check: