

Introduction to Sensible Calculus: A Thematic Approach



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Martin Flashman

Professor of Mathematics
Humboldt State University

flashman@humboldt.edu

<http://users.humboldt.edu/flashman>

Day by Day Outline (Rev'd 6-26)

0. Sunday: Basic Themes Plus ...

- Mapping Diagrams
- Technology (Winplot and Geogebra)

I. Monday: Making Sense of the Derivative.

II. Tuesday: More on the Derivative

III. Wednesday: DE's, Approximation and The Fundamental Theorem of Calculus

IV. Thursday: More on the FT, DE's, Models and Estimations

V. Friday: Frontiers-Making Sense of Taylor Theory, Probability.

Brief Review:

Concept and Pedagogical Principles

- Themes of differential equations and estimation, using modeling.
 - "...everything in a calculus course can be related to the study of differential equations."
 - "...estimation is valuable for both numerical and conceptual development."
- The consistent use of interpretations provides meaning for calculus concepts and helps develop habits of the mind.
 - Present examples of models or arguments before more general applications and proofs.
 - Form a foundation for later learning of concepts, language, and notation.
 - understand the specific and particular in experience and then **unify, generalize, ..., abstract.**

Continuing from Last Class

Sensible Calculus: Two Forms of the Fundamental Theorem of Calculus

Evaluation Form

If f is continuous and $F'(x) = f(x)$ for all x ... then

$$\int_a^b f(x) dx = F(b) - F(a).$$

Derivative Form (Barrow's Theorem)

If f is continuous and $G(t) = \int_a^t f(x) dx$ then

G is a differentiable function and $G'(t) = f(t)$.

FT of Calculus

Objective & Key Ideas

Two Key Ideas:

- When x is close to a , $f(x)$ is approximately equal to a linear function,
$$f(a) + f'(a)(x - a).$$

(The Differential/Linear Estimator)

- As long as f is a sufficiently well behaved function there is some c between a and b where

$$f(b) - f(a) = f'(c)(b - a).$$

(The Mean Value Theorem - MVT)

Sensible Proofs of FT (Evaluation)

- Use an extended Euler Sum to estimate $\int_a^b f(x) dx$ then use the Mean Value Theorem for each subinterval and watch the sum telescope!
 - For each interval choose c_k where
$$f(c_k) * \Delta x = F'(c_k) * \Delta x = F(x_{k+1}) - F(x_k)$$
SO the sum "telescopes" to
$$F(b) - F(a).$$
 - Interpret with motion and geometry.

Sensible Proofs of FT (Evaluation)

Use FT of C Derivative Form to justify FT of C Evaluation.

- $G'(t) = F'(t)$ so $G(t) = F(t) + C$ for some C .

- $G(a) = 0 = F(a) + C$ so $C = -F(a)$

- So

$$\int_a^b f(x) dx = G(b) = F(b) + C = F(b) - F(a)$$

Session V : Frontiers-Probability, Economics, ...

We complete our introduction to making calculus sensible by a consideration of some frontiers and results that provide both motivation and consolidation for the first year experience with calculus.

Focus Themes for Taylor Theory:
Estimation, Differential Equations, Models

Focus on estimating a growth model
with a differential equation:

$$P'(x) = P(x), \quad P(0) = 1.$$

- Solution is already treated
 - with estimation by Euler's method.
 - "Exactly":

$$P(x) = e^x$$

Focus Themes for Taylor Theory:
Estimation, Differential Equations, Models

$$P'(x) = P(x), \quad P(0) = 1.$$

- Estimation of the solution: Use the polynomial of degree n that best matches the differential equation.
- Determine estimate of error for estimating
 - e
 - $\int_0^1 e^{-x^2} dx$

Sample Exercises

1. Use the Taylor polynomial for e^x of degree 4 to estimate the following:
(a) e^2 (b) e^3 (c) $e^{0.5}$ (d) e^{-1} (e) $e^{3.14}$. [Spreadsheet helper supplied.]
2. Estimate e using the Taylor polynomial of degree n where n is (a) 6 (b) 7 (c) 8 (d) 10.
In each estimate discuss the size of the error term R_n . [Spreadsheet helper.]
3. What value of n should be used so that the Taylor polynomial of degree n will give an estimate of e that is within .000001 of the exact value of e ? Explain your result.
4. Use the Taylor polynomial for e^x of degree 5 to estimate $\int_0^1 e^{-t^2} dt$.
Discuss the error in this approximation.

Sensible Calculus: Evolving Taylor Theory

- IX.A Taylor Theory for e^x
- IX.B MacLaurin Polynomials and Taylor Theory
- IX.C MacLaurin Polynomials: How to Find Them
- IX.D Taylor Polynomials

Session V : Frontiers-Probability

What are some frontiers for the first year experience with calculus?

- Probability and Calculus
 - Choose Darts, NOT Dice
 - Start with distributions, not density
 - Make sense of calculus with probability
 - Make sense of probability with calculus

Session V : Frontiers-Economics

What are some frontiers for the first year experience with calculus?

- **Economics and Calculus**
 - Choose Micro before Macro
 - Start with margins
 - Make sense of calculus with economics
 - Make sense of economics with calculus

Session V : Frontiers-History

What are some frontiers for the first year experience with calculus?

- History and Calculus
 - Choose (original) sources before biographical sketches
 - Start with pre-Newton/Leibniz
 - Euclid, Archimedes, Galileo, Kepler, Descartes, Fermat, Barrow, ...
 - Make sense of calculus with history
 - Make sense of history with calculus

Session V : Frontiers-Probability More Detail...

- Probability and Calculus
 - Darts, NOT Dice
 - Start with distributions, not density
 - Make sense of calculus with probability
 - Make sense of probability with calculus

End of Session V

- Questions?

**Thanks
The End!**



**Still have questions?
Comments?**

e-mail them to me:

flashman@humboldt.edu

**Thanks
The End!**



Questions?

flashman@humboldt.edu

<http://www.humboldt.edu/~mef2>