Using Mapping Diagrams to Understand Linear Functions

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Using Mapping Diagrams to Understand Linear Functions Links:

http://users.humboldt.edu/flashman/Exeter/Exeter.MD.LINKS.html

Background Questions

- Are you familiar with Mapping Diagrams?
- Have you used Mapping Diagrams to teach functions?
- Have you used Mapping Diagrams to teach content besides function definitions?

Mapping Diagrams

A.k.a.

Function Diagrams

Dynagraphs

Main Resource

- Mapping Diagrams from A(lgebra) B(asics) to C(alculus) and D(ifferential) E(quation)s. A Reference and Resource Book on Function Visualizations Using Mapping Diagrams (Preliminary Sections- NOT YET FOR publication)

- http://users.humboldt.edu/flashman/MD/section-1.1VF.html

Linear Mapping diagrams

We begin a more detailed introduction by a consideration of linear functions :

$$\text{“ } y = f(x) = mx + b \text{ ”}$$

Show Worksheet *now.*

Do Problem 1

Prob 1: Linear Functions -Tables

x	5 x - 7
3	
2	
1	
0	
-1	
-2	
-3	

Complete the table.

$x = 3,2,1,0,-1,-2,-3$

$f(x) = 5x - 7$

$f(0) = \_\_\_\_?$

For which x is $f(x) > 0$?

Linear Functions: Tables

x	5 x – 7
3	8
2	3
1	-2
0	-7
-1	-12
-2	-17
-3	-22

Complete the table.

$x = 3,2,1,0,-1,-2,-3$

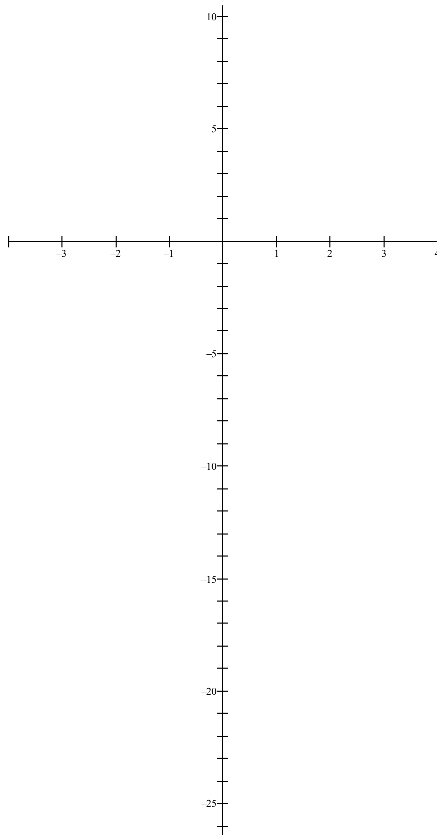
$f(x) = 5x - 7$

$f(0) =$ ____?

For which x is $f(x) > 0$?

Linear Functions: On Graph

Plot Points (x , 5x - 7):



X	5 x - 7
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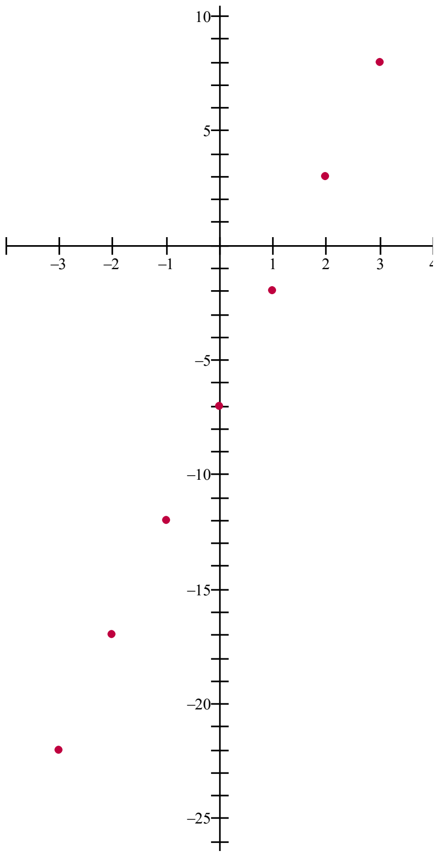
Linear Functions: On Graph



Connect Points

$(x\ ,\ 5x - 7)$:

X	5 x - 7
3	8
2	3
1	-2
0	-7
-1	-12
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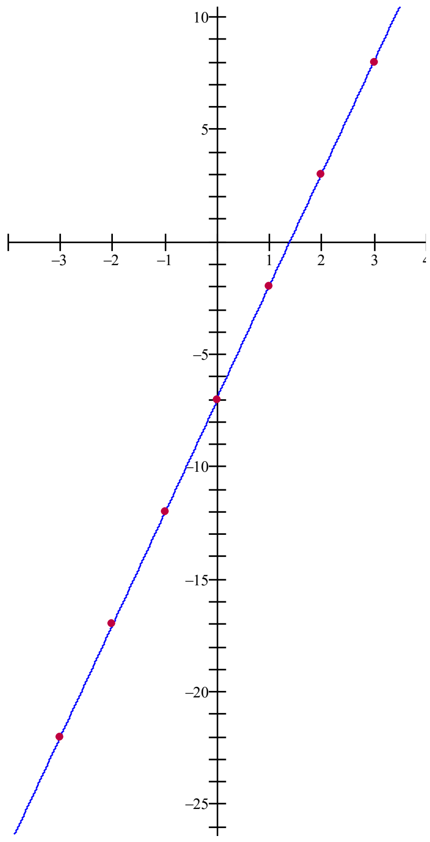
-3	-22

Linear Functions: On Graph



Connect the Points

X	5 x - 7
3	8
2	3
1	-2
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-2	-17
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Linear Functions: Mapping diagrams Visualizing the table.

- Connect point x on left axis to the point 5x – 7 on the right axis.

X	5 x – 7
3	8
2	3
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Linear Functions: Mapping diagrams Visualizing the table.

- Connect point x on left axis to the point 5x – 7 on the right axis.

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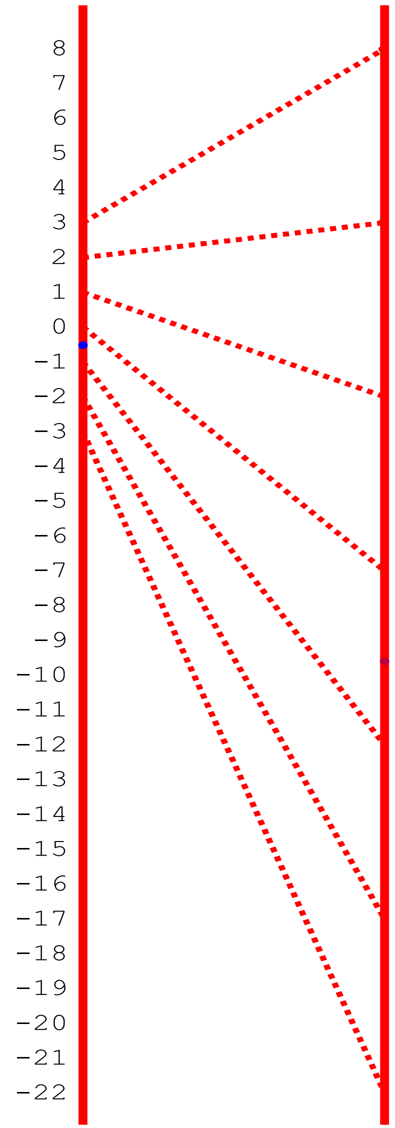


Technology Examples

- Excel example
- Geogebra example

Visualizing Linear Functions

- Linear functions are both necessary, and understandable- even without considering their graphs.
- There is a sensible way to visualize them using "mapping diagrams."
- Examples of important function features (like rate and intercepts) can be illustrated with mapping diagrams.
- Activities for students engage understanding both for function and linearity concepts.
- Mapping diagrams can use simple straight edges as well as technology.

Simple Examples are important!

- $f(x) = x + C$ Added value: C
- $f(x) = mx$ Scalar Multiple: m

  Interpretations of m:
  - slope
  - rate
  - Magnification factor
  - $m > 0$: Increasing function
  - $m = 0$: Constant function
  - $m < 0$: Decreasing function

Simple Examples are important!

$f(x) = mx + b$ with a mapping diagram --

Five examples:

Back to Worksheet Problem #2

- Example 1: $m =-2$; $b = 1$: $f(x) = -2x + 1$
- Example 2: $m = 2$; $b = 1$: $f(x) = 2x + 1$
- Example 3: $m = \frac{1}{2}$; $b = 1$: $f(x) = \frac{1}{2} x + 1$
- Example 4: $m = 0$; $b = 1$: $f(x) = 0 x + 1$
- Example 5: $m = 1$; $b = 1$: $f(x) = x + 1$

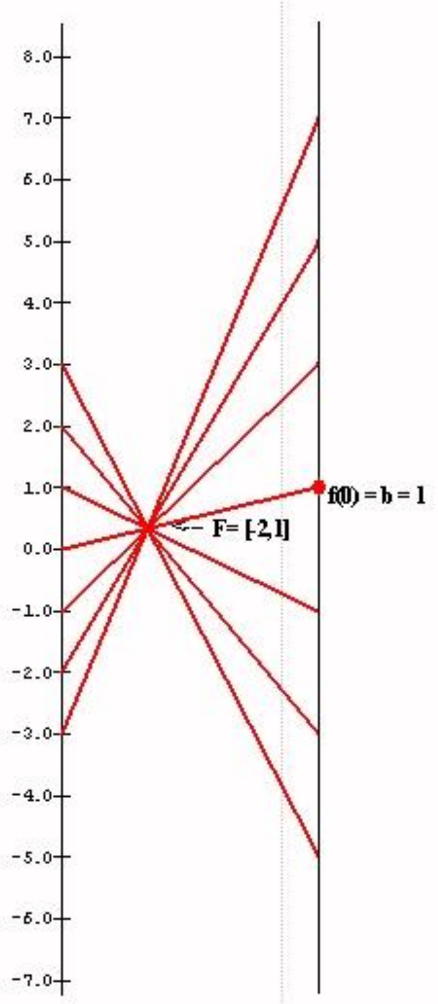
Visualizing $f(x) = \boldsymbol{mx} + \boldsymbol{b}$ with a mapping diagram -- Five examples:

Example 1: m = -2; b = 1

f (x) = -2x + 1

- Each arrow passes through a single point, which is labeled **F = [- 2,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, -2*x* + 1,

  which corresponds to the linear function's value for the point/number, *x*.



Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

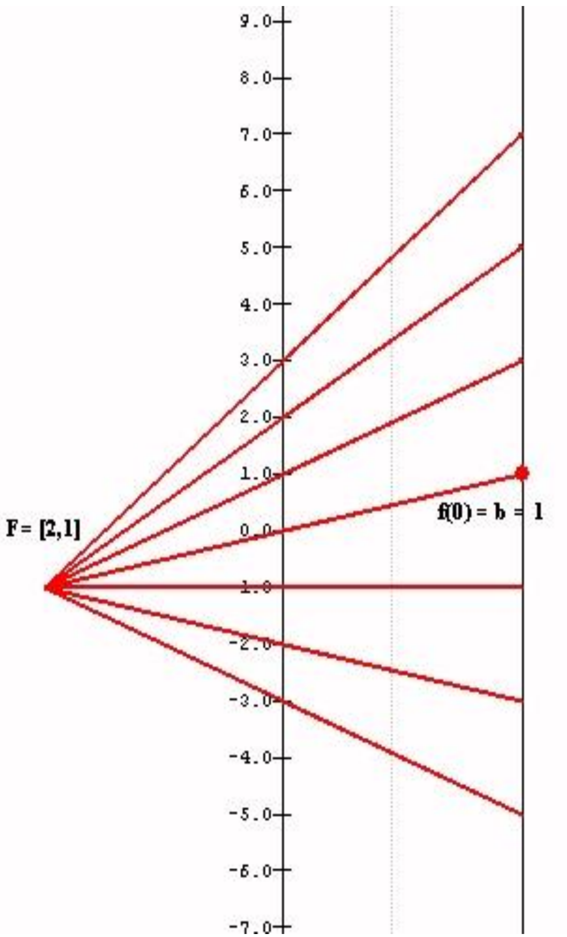
Example 2: $m = 2$; $b = 1$
$f(x) = 2x + 1$

Each arrow passes through a single point, which is labeled

F = [2,1].

- The point **F** completely determines the function *f*.
  - **given a point** / number, *x*, on the source line,
  - there is a **unique arrow passing through F**
  - **meeting** the target line at **a unique point** / number, $2x + 1$,

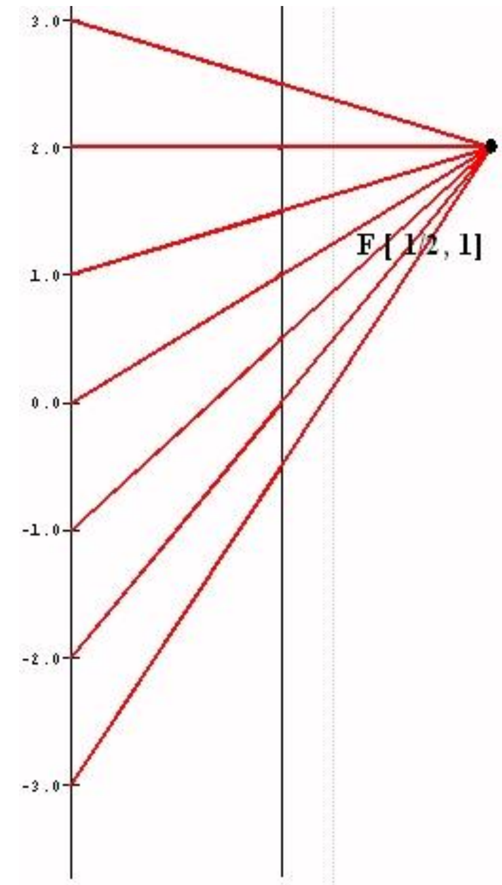
which corresponds to the linear function's value for the point/number, *x*.



Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

- **Example 3: *m* = 1/2; *b* = 1**
  $f(x) = \frac{1}{2}x + 1$
- Each arrow passes through a single point, which is labeled **F = [1/2,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $\frac{1}{2}x + 1$,

    which corresponds to the linear function's value for the point/number, *x*.

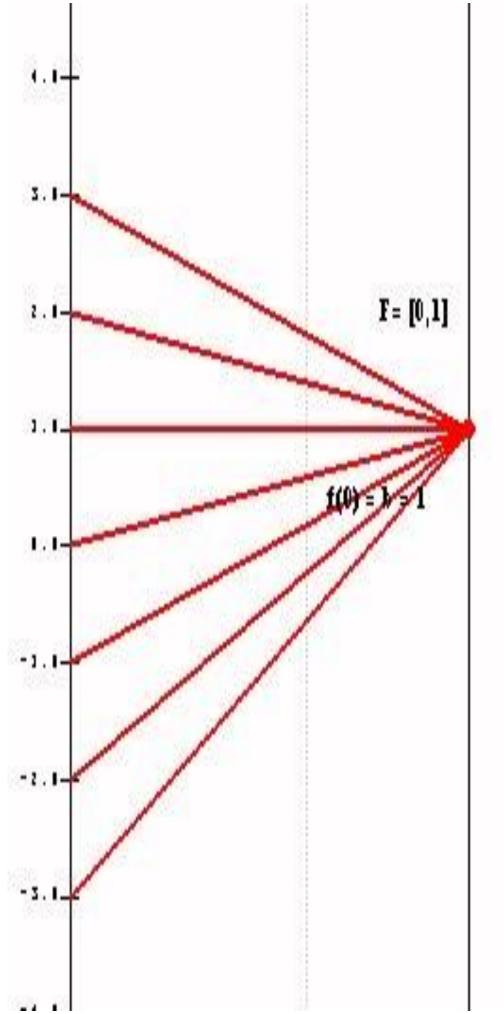


Visualizing $f(x) = \boldsymbol{m}x + \boldsymbol{b}$ with a mapping diagram -- Five examples:

- **Example 4: $\boldsymbol{m} = 0$; $\boldsymbol{b} = 1$**
  $f(x) = 0\,x + 1$

- Each arrow passes through a single point, which is labeled **F = [0,1]**.
  - The point **F** completely determines the function *f*.
    - **given a point** / number, *x*, on the source line,
    - there is a **unique arrow passing through F**
    - **meeting** the target line at **a unique point** / number, $f(x)=1$,

    which corresponds to the linear function's value for the point/number, *x*.

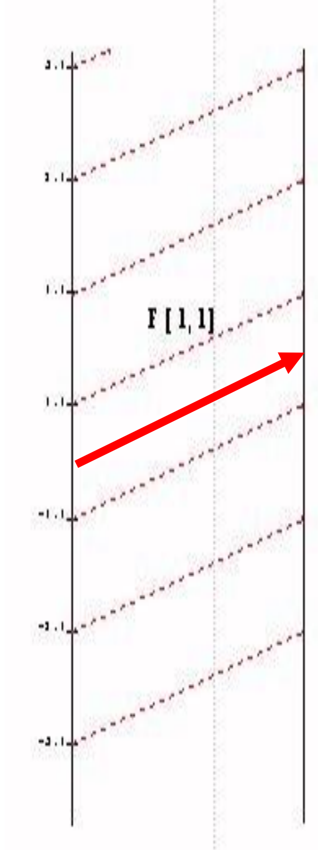


Visualizing $f\ (x)\ =\ \boldsymbol{m}x\ +\ \boldsymbol{b}$ with a mapping figure -- Five examples:

_**Example 5:** $\boldsymbol{m}\ = \mathbf{1};\ \boldsymbol{b}\ =\ \mathbf{1}$

$$f\ (x)\ = 1x\ +\ \mathbf{1}$$

- Unlike the previous examples, in this case it is not a single point that determines the mapping figure, but the single arrow from 0 to 1, which we designate as $F[1,1]$
- It can also be shown that this single arrow completely determines the function. Thus, given a point / number, x, on the source line, there is a unique arrow passing through $\boldsymbol{x}$ ***parallel to*** $F[1,1]$ meeting the target line a unique point / number, $x\ +\ 1$, which corresponds to the linear function's value for the point/number, x.
  - The single arrow completely determines the function f.
    - **given a point** / number, x, on the source line,
    - there is a **unique arrow** through $\boldsymbol{x}$ ***parallel to*** $F[1,1]$
    - **meeting** the target line at **a unique point** / number, $x\ +\ 1$, which corresponds to the linear function's value for the point/number, x.

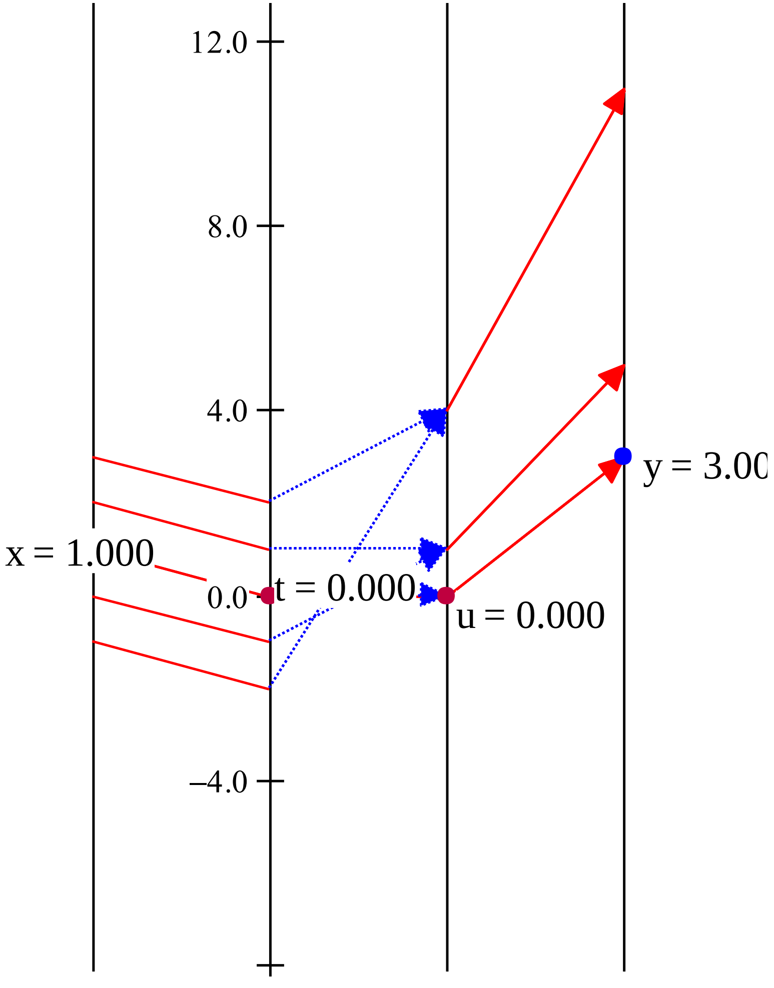


How Linear Functions Fit into Other Functions: Quadratic Example
Will be reviewed at end. ☺

$g(x) = 2\,(x-1)^2 + 3$

Steps for g:

1. **Linear:** Subtract 1.
2. **Square result.**
3. **Linear:** Multiply by 2 then add 3.



Function-Equation Questions

with linear focus points (Problem 3)

- Solve a linear equation:

  $2x+1 = 5$

  - Use focus to find x.

Function-Equation Questions

with linear focus points (Problem 4)

Suppose f is a linear function with $f(1) = 3$ and $f(3) = -1$.

- Use focus to find $f(0)$.
- Use focus to find x where $f(x) = 0$.

More on Linear Mapping diagrams

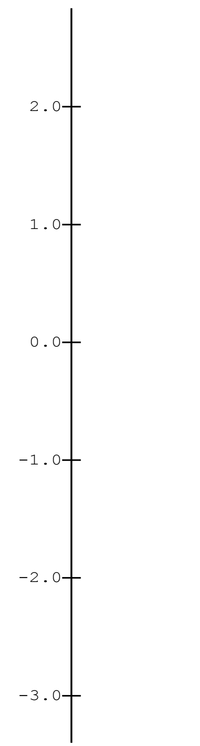
We continue our introduction to mapping diagrams by a consideration of the **composition of linear functions**.

Do Problem 5

Problem 5: Compositions are keys!

An example of composition with mapping diagrams of simpler (linear) functions.

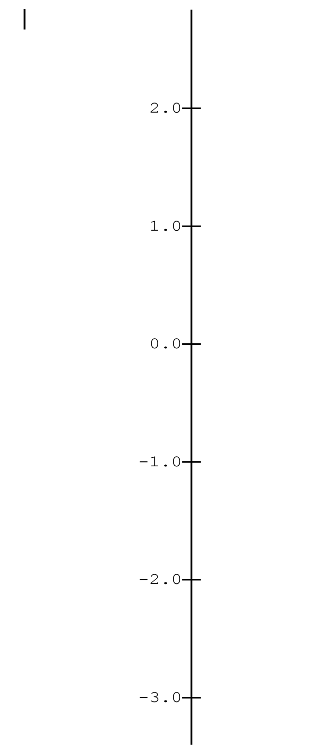
- $g(x) = 2x$; $h(x)=x+1$
- $f(x) = h(g(x)) = h(u)$
  where $u = g(x) = 2x$
- $f(x) = (2x) + 1 = 2x + 1$
  $f(0) = 1 \quad m = 2$



Compositions are keys!

All Linear Functions can be understood and visualized as compositions with mapping diagrams of simpler linear functions.

- $f(x) = 2x + 1 = (2x) + 1$:
  - $g(x) = 2x$; $h(u)=u+1$
  - $f(0) = 1$ $m = 2$



Compositions are keys!

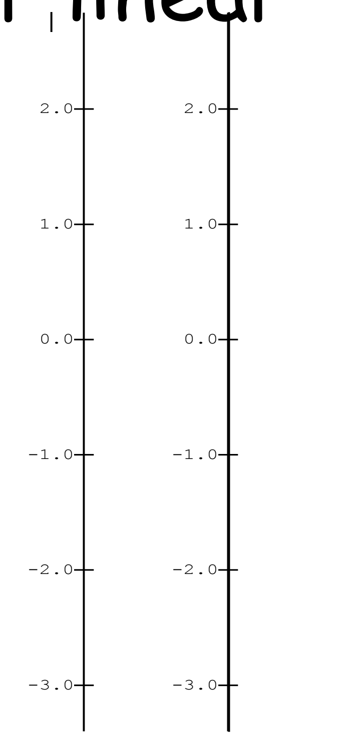
All Linear Functions can be understood and visualized as compositions with mapping diagrams of simpler linear functions.

Point Slope Example:

$f(x) = 2(x-1) + 3$

$g(x)=x-1$ $h(u)=2u$; $k(t)=t+3$

- $f(1)= 3$ slope = 2



Questions for Thought

- For which functions would mapping diagrams add to the understanding of composition?
- In what other contexts are composition with "x+h" relevant for understanding function identities?
- In what other contexts are composition with "-x" relevant for understanding function identities?

Inverses, Equations and Mapping diagrams

- Inverse: If $f(x) = y$ then $f^{-1}(y) = x$.
- So to find $f^{-1}(b)$ we need to find any and all x that solve the equation
$$f(x) = b.$$
- How is this visualized on a mapping diagram?
- Find b on the target axis, then trace back on any and all arrows that "hit" b.

Mapping diagrams and Inverses

Inverse linear functions:

Classroom Activity

- Use transparency for mapping diagrams-
  - Copy mapping diagram of f to transparency.
  - Flip the transparency to see mapping

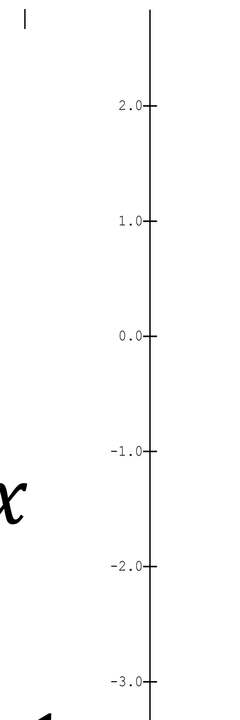
  diagram of inverse function $g = f^{-1}$.

  ("before or after")

  $$f(g(b)) = b; \quad g(f(a)) = a$$

- Example i: $g(x) = 2x$; $g^{-1}(x) = \frac{1}{2}x$
- Example ii:

  $$h(x) = x + 1;\ h^{-1}(x) = x - 1$$

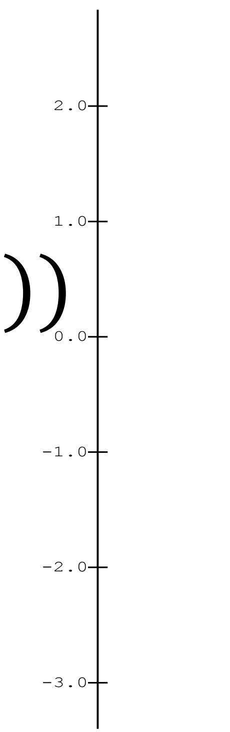


Mapping diagrams and Inverses

Inverse linear functions:

- socks and shoes with mapping diagrams
- $g(x) = 2x;\ g^{-1}(x) = \frac{1}{2}x$
- $h(x) = x + 1;\ h^{-1}(x) = x - 1$
- $f(x) = 2x + 1 = (2x) + 1 = h(g(x))$
- $g(x) = 2x;\ h(u) = u + 1$
  - The inverse of f:

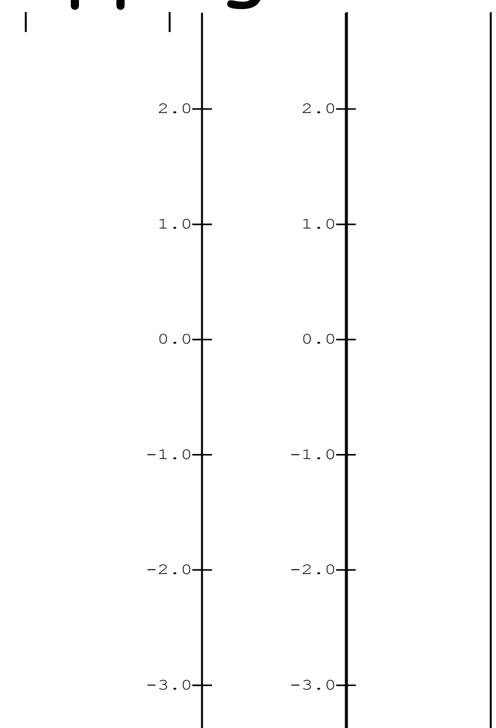
$$f^{-1}(x) = g^{-1}(h^{-1}(x)) = \frac{1}{2}(x - 1)$$



Mapping diagrams and Inverses

Inverse linear functions:

- “socks and shoes” with mapping diagrams
- $f(x) = 2(x-1) + 3$:
  - $g(x) = x - 1$
  - $h(u) = 2u$
  - $k(t) = t + 3$
  - The inverse of f:



$$f^{-1}(x) = \frac{1}{2}(x-3) + 1$$

Questions for Thought

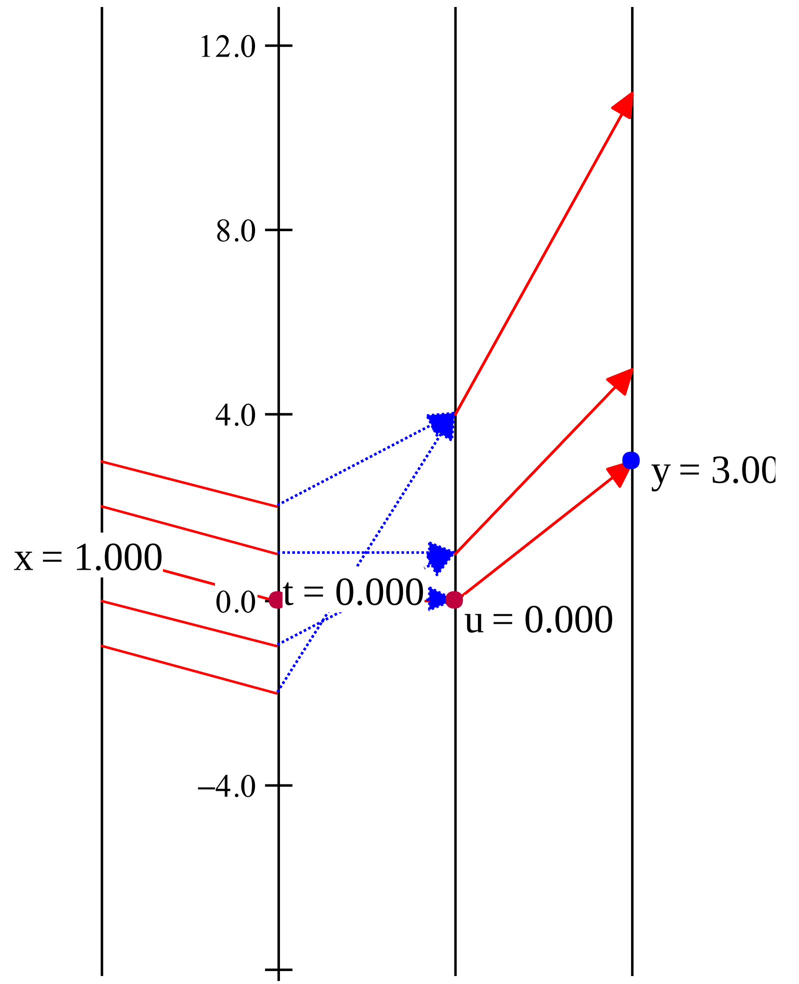
- For which functions would mapping diagrams add to the understanding of inverse functions?
- How does “socks and shoes” connect with solving equations and justifying identities?

Closer: Quadratic Example Continued. ☺

$g(x) = 2\,(x-1)^2 + 3$

Steps for *g*:

1. **Linear:** Subtract 1.
2. **Square result.**
3. **Linear:** Multiply by 2 then add 3.



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Thanks
The End!

☺

Questions?

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