

Section 1.9 Theorems Related to the Matrix of a Linear Transformation.

Theorem 10:

Let $T: \mathbb{R}^n \rightarrow \mathbb{R}^k$ be a linear transformation. Then there is a unique matrix A so that $T(\mathbf{x}) = A\mathbf{x}$ for all \mathbf{x} in \mathbb{R}^n .

Theorem 11:

Let $T: \mathbb{R}^n \rightarrow \mathbb{R}^k$ be a linear transformation.. Then T is one to one if and only if the equations $T(\mathbf{x}) = \mathbf{0}$ has only the trivial solution.

Theorem 12:

Let $T: \mathbb{R}^n \rightarrow \mathbb{R}^k$ be a linear transformation and let A denote the standard matrix for T (as in Theorem 10). Then:

- a. T is onto if and only if the columns of A span \mathbb{R}^k .
- B. T is one to one if and only if the columns of A are linearly independent.